On-Line Identification of Discrete Event Systems: a Case Study*

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Abstract—The paper analyses an on-line identification strategy for Discrete Event Systems (DES) using Interpreted Petri Nets (IPN). The identifier stores a sequence of events and the corresponding output symbols and applies a recursive algorithm providing an IPN modeling the DES. Moreover, the identification procedure is based on the solution of an integer linear programming problem. In addition, we investigate on the conditions that lead to determine an IPN modeling the DES dynamics without error. Finally, simulation and analysis of a case study show the efficiency of the strategy.

Index terms: Discrete event systems, Petri nets, identification, integer linear programming.

I. INTRODUCTION

SYSTEM identification deals with choosing mathematical models from a known model set to characterize the input-output behavior of an unknown system from finite data [14]. The problem of identifying the language of a Discrete Event System (DES) from finite data has been explored in a pioneering contribution by Gold [6], who shows that the problem of finding a finite automaton accepting positive samples of a regular language is NP complete.

Petri Nets are a standard model for the study of DES and Cabasino et al. [1] propose a linear algebraic approach for the identification of a PN from the knowledge of the DES language, i.e., the set of strings generated by the set of transitions. The authors suppose that the language is finite, prefix-closed and that the length of the longest string it contains is known. In addition, the approach solves the identification problem by Integer Linear Programming (ILP), that is applied to λ-free labeled nets [10]. Moreover, Meda et al. [8, 9] use an incremental synthesis approach to identify an Interpreted Petri Net (IPN) modeling a DES on the basis of the partial knowledge of input and output symbols. More precisely, IPN are an extension of PN that allows representing the output signals generated when a marking is reached and associating labels with transitions and events.

This paper addresses the identification problem of DES on the basis of an on-line identification approach [3]. The identifier records a sequence of events and the resulting output response sequence. Hence, on the basis of such available knowledge of the system, the on-line procedure chooses the set of places and the output function defining the relation between places and output symbols and imposes a set of suitable linear constraints. Moreover, a metric [12] is defined as an indicator of the size of the IPN and an ILP problem is established in order to minimize the selected metric and to obtain an IPN modeling the DES. The present paper extends the previous work [3] in two respects: first, we investigate on the conditions that lead to determine an IPN modeling the DES dynamics without error; second, we simulate and analyze a case study to show the strategy efficiency.

The paper is organized as follows. Section II summarizes several basic definitions and notations related to PN and IPN. Moreover, Section III defines the DES identification problem and, after recalling the on-line identification technique, presents some results characterizing the observed word. In addition, Section IV presents the case study simulation and analysis. Finally, Section V draws the conclusions.

II. BASIC DEFINITIONS ON PN AND IPN

This section provides some basic definitions on PN and IPN used in this paper [2, 10, 11].

A. Petri Nets

Definition 1: A PN is a bipartite digraph described by the four-tuple $P^N=(P, T, Pre, Post)$ where $P$ is a set of places with cardinality $m$, $T$ is a set of transitions with cardinality $n$, $Pre: P \times T \to N$ and $Post: P \times T \to N$ are the pre- and post-incidence matrices respectively. More precisely, for each $p \in P$ and $t \in T$ element $Pre(p,t)$ ($Post(p,t)$) is equal to a natural number indicating the arc multiplicity if an arc going from $p$ to $t$ (from $t$ to $p$) exists, and it equals 0 otherwise. Note that $N$ is the set of non-negative integers.

The $m \times n$ incidence matrix of the net is defined as follows:

$$C(p,t)=Post(p,t)-Pre(p,t).$$

(1)

Given a PN, for pre- and post-sets we use the dot notation, e.g. $p\cdot T = \{ t \in T : Pre(p,t) > 0 \}$.

The state of a PN is given by its current marking, which is a mapping $M: P \to N$, assigning to each place of the net a non-negative number of tokens. A PN system $\langle P^N, M_0 \rangle$ is a net $P^N$ with an initial marking $M_0$. A transition $t \in T$ is enabled at a marking $M$ if and only if (iff) for each $p \in \bullet t$, it holds:

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\[ M(\rho) \cong \text{Pre}(\rho, t_j) \]  
and we write \( M(t_j) \) to denote that \( t_j \in T \) is enabled at a marking \( M \). When fired, \( t_j \) produces a new marking \( M' \), denoted by \( M(t_j) = M' \) that is computed by the PN state equation:

\[
M' = M + C \cdot t_j ,
\]

with \( t_j \) the \( n \)-dimensional firing vector corresponding to the \( j \)-th canonical basis vector. Let \( \sigma = [t_1, t_2, \ldots, t_k] \) be a sequence of transitions (or firing sequence) and let its length \( k \) be the number of transitions that \( \sigma \) contains. If a transition \( t \in T \) appears in \( \sigma \), we write \( t \in \sigma \). In addition, we denote by \( |\sigma| \) the number of occurrences of transition \( t \) in \( \sigma \). Moreover, we write \( M[\sigma] = M' \) to denote that the sequence of enabled transitions \( \sigma \) may fire at \( M \) yielding \( M' \). The firing vector of \( \sigma \) is denoted by \( \hat{\sigma} \). A marking \( M \) is said reachable from \( \langle PN, M_0 \rangle \) iff there exists a firing sequence \( \sigma \) such that \( M[\sigma] = M' \). The set of all markings reachable from \( M_0 \) defines the reachability set of \( \langle PN, M_0 \rangle \), denoted by \( R(PN, M_0) \).

**B. Interpreted Petri Nets**

A language may be a formal way describing the behavior of a DES. The event set \( E \) is viewed as an alphabet and \( LCE^* \) is the set of all words (event sequences) generated by the system. Moreover, if a PN is used to model the DES, the system events are associated with transitions and the conditions that govern the DES are the place markings. However, each transition of \( T \) does not necessarily correspond to a distinct event from the set \( E \). Moreover, the system state could be only partially observed from the system output. Hence, this leads to the introduction of IPN [10, 11] that allow representing the output signals generated when a marking is reached and associating labels with transitions and events.

**Definition 2:** An IPN is the eight-tuple \( Q=(P, T, \text{Pre}, \text{Post}, E, Y, \lambda, \Psi) \) where: \( \text{Pre}=(P, T, \text{Pre}, \text{Post}) \) is a PN; \( E=\{e\} \) is the event set of cardinality \( \nu \); \( Y=\{y\} \) is the set collecting the output symbols associated with places; \( \lambda : T \rightarrow E \) is the transition labeling function that assigns to each transition \( t \in T \) a symbol \( e \in E \). The labeling function is \( \lambda \)-free according to [10], i.e., the same label \( e \in E \) may be associated to more than one transition while no transition may be labeled with the empty string \( \epsilon \). Finally, \( \Psi : R(PN, M_0) \rightarrow \{Y\}^q \) is an output function and \( q \) is the number of available outputs associated with places.

An IPN system \( \langle Q, M_0 \rangle \) is an IPN \( Q \) with initial marking \( M_0 \). The following remarks and definitions specify and characterize the IPN considered in this paper.

**Remark 1.** We assume that the output set of symbols is \( Y=N \) and \( \Psi \) is defined as a \( q \times n \) matrix \( \Psi = \{\Psi(i, j)\} \). Denoting the \( i \)-th row vector \( \Psi(i, \cdot) \) equal to the transpose of the \( j \)-th canonical vector, function \( \Psi \) imposes a correspondence between the \( i \)-th output symbol and the \( j \)-th place. Hence, \( y=\Psi M \) is the measured marking vector of dimension \( q \).

Moreover, the present definition of IPN assumes that all transitions can be observed by the output of the system.

**Definition 3:** Given an IPN system \( \langle Q, M_0 \rangle \), a place \( p \in P \) is said measurable if \( \exists j \in \{1, \ldots, q\} \) such that \( \Psi(i, j)=1 \).

We denote \( P_m \) the set of measurable places and \( P_{nm} \) the set of non measurable places, with \( P_m \cap P_{nm} = \emptyset \). The partition of the place set induces a corresponding decomposition on the marking vectors and on the incidence matrix. Hence, each marking \( M \in R(PN, M_0) \) and matrix \( C \) can be rewritten as follows:

\[
M = \begin{bmatrix} M^m \\ M^{nm} \end{bmatrix}, \quad C = \begin{bmatrix} C^m \\ C^{nm} \end{bmatrix},
\]

where \( M^m \) (\( C^m \)) is a sub-vector (sub-matrix) of dimension \( q \) \((q \times n)\) corresponding to measurable places and \( M^{nm} \) (\( C^{nm} \)) is a sub-vector (sub-matrix) of dimension \( m-q \) \(((m-q) \times n)\).

Now, a definition about determinism in IPN [4] is recalled as a behavioral property depending on the reachable set.

**Definition 4:** The PN system \( \langle Q, M_0 \rangle \) with labeling function \( \lambda \) is said deterministic if \( \forall M \in R(PN, M_0) \) and \( \forall t_1, t_2 \in T : t_1 \neq t_2, \lambda(t_1) = \lambda(t_2) \) it holds \( M[t_1] \rightarrow M[t_2] \), i.e., any two transitions with the same associated symbol cannot have the same input places.

The set of all firing sequences in \( \langle Q, M_0 \rangle \) is denoted by \( L(Q, M_0) = \{ \sigma \in T^* \mid M_0[\sigma] > 0 \} \) and for each \( \sigma \in L(Q, M_0) \) with \( \sigma = [t_1, t_2, \ldots, t_k] \), the evolution of the net is the following:

\[
M_0[t_1], M_0[t_2], \ldots, M_0[t_k] \rightarrow M_{t_k}.
\]

We denote by \( w \) the word of events associated to the sequence \( \sigma \) with \( w=\lambda(\sigma) \), using the extended form of the transition labeling function \( \lambda : T^* \rightarrow E^* \) in the usual manner. Moreover, the empty string \( \epsilon \) is associated with the word of null length. The language generated by the IPN system \( \langle Q, M_0 \rangle \) is:

\[
L^F(Q, M_0) = \{ w \in L \mid \lambda(\sigma) = w, \sigma \in T^*, M_0[\sigma] > 0 \}.
\]

### III. The DES Identification Problem

This section defines the DES Identification Problem (IP) and recalls a technique to obtain a deterministic IPN modeling the DES [3]. Let us introduce the following PN set:

\[
D = \{ PN=(P, T, \text{Pre}, \text{Post}) : \text{Pre} \in N^{\text{Post}}, \text{Post} \in N^{\text{Pre}} \}.
\]

The DES identification problem is stated as follows.
Definition 5: Let us consider a DES and let \( E \) be the set of events of the DES of cardinality \( v \). Moreover, let \( w \) be an event sequence generated by the DES, \( Y=N \) the set of output symbols and \( y \in N^q \) the output vector of dimension \( q \). Chosen a set of places \( P \) of cardinality \( m \geq q \), a set of transitions \( T \) of cardinality \( m \geq q \) and an output function \( \Psi \), the Identification Problem consists in determining a deterministic IPN system \( \langle Q, M_0 \rangle \) with labeling function \( \lambda : Q \rightarrow (P, E, Y, \lambda, \Psi), P \subseteq D \) and \( M_0 \in N^m \) such that \( y \in L^k(Q, M_0) \).

A. The Identification Technique

We consider a DES whose dynamics is driven by the event set \( E \) and provides the output vector \( y \in N^q \). On the basis of the available knowledge of the system, we choose a set of places \( P \) of cardinality \( m \geq q \) and an output function \( \Psi \), imposing the relation between places and output symbols.

We denote by \( T'=\{ t_i \in T | \lambda(t_i)=\varepsilon \} \) the set of transitions associated with the same event \( \varepsilon \) and \( z=\text{Card}(T') \), where \( \text{Card}(.) \) indicates the cardinality of set (.).

Consequently, if we observe a word \( w = \varepsilon \alpha_1 \varepsilon \alpha_2 ... \varepsilon \alpha_k \) then we infer \( \sigma = \varepsilon \alpha_1 \varepsilon \alpha_2 ... \varepsilon \alpha_k \) with \( t_{alpha_k} \in T'. \) Moreover, we record the sequence \( w \) of events and the corresponding sequence of output vectors \( y_{a_i} \) for \( i=1, . . . , k \) that are collected in the \((q \times k)\) matrix \( Y \) with \( Y(i, .) = y_{a_i} \). Denoting by

\[ M_0 = \begin{bmatrix} \Phi_0 & M_a & \Gamma_{0M} \end{bmatrix} \]

where \( \Phi_0 \) is the matrix of dimensions \( m \times m \) with each element being 1 (0). By \( \Phi_0 \) we denote the \( \Phi_0 \) behavior according to the observed sequence \( Y \) and \( v \) the output function \( \Psi \). In addition, at each event occurrence the IA adds the transition corresponding to that event to the firing sequence \( \sigma \) and stores the corresponding DES output \( y_{a_i} \). Hence, the outputs of the algorithm are the incidence matrix \( \hat{\Phi} = \hat{\Phi} - \hat{\Phi} \), the initial marking \( M_0^G \) and the labeling function \( \lambda \). Summing up, the IA output is an IPN system \( \langle Q, M_0^G \rangle \) modeling the DES behavior according to the observed sequence \( \sigma \). A detailed discussion on the IA can be found in [3].

Theorem 1: If an IPN system \( \langle Q, M_0 \rangle \), deterministic over the labeling function \( \lambda \), satisfies the set of linear algebraic constraints \( \xi (w, Y, \lambda, n) \), then it is a solution of the IP.

In general, there is not only one IPN satisfying the constraint set \( \xi (w, Y, \lambda, n) \). Hence, to select an IPN and an initial marking, we introduce a metric as a linear function \( \phi : D \times N^m \rightarrow N \) and we choose a PN in \( D \) and a marking in \( N^m \) that satisfy the constraint set and minimize the metric \( \phi \).

Proposition 1: A solution of the IP is obtained solving the following ILP problem:

\[
\min \phi(Pre, Post, M_0) = \bar{1}_{1xm} \cdot (Pre + Post) \cdot \bar{1}_{nx1} + \bar{1}_{1xm} \cdot M_0
\]

s.t. \( \xi (w, Y, \lambda, n) \). (9)

To evaluate the computational complexity of the optimization problem defined by (9), it is easy to infer [7] that in the worst case the number of unknowns is \( \theta = m(2n+1+k) \), i.e., \( \theta \) is linear with the number of places, of transitions and with the firing sequence length \( k \).

The Identification Algorithm: The following Identification Algorithm (IA) is applied on-line to obtain an IPN modeling the DES generating language \( L \) with the recursive solution of the defined ILP. At each event occurrence, the procedure refines the IPN and, after the observation of a sequence \( \sigma \), it provides an IPN system \( \langle \hat{Q}, \hat{M}_0^G \rangle \) that tends to approach the IPN system \( \langle Q, M_0 \rangle \) such that \( L^k(Q, M_0) = L \). When the algorithm is invoked, the following items are assumed known: the place set \( P \) and its cardinality \( m \), the output symbol set \( Y \), the number \( q \) of the available outputs, the event set \( E \) and its cardinality \( v \), the output function \( \Psi \). In addition, at each event occurrence the IA adds the transitions corresponding to that event to the firing sequence \( \sigma \) and stores the corresponding DES output \( y_{a_i} \). Hence, the outputs of the algorithm are the incidence matrix \( \hat{\Phi} = \hat{\Phi} - \hat{\Phi} \), the initial marking \( M_0^G \) and the labeling function \( \lambda \). Summing up, the IA output is an IPN system \( \langle \hat{Q}, \hat{M}_0^G \rangle \) modeling the DES behavior according to the observed sequence \( \sigma \). A detailed discussion on the IA can be found in [3].

1. Initializing the algorithm variables.
   \[ w := \varepsilon ; \; \sigma := \varepsilon ; \; i := 1, \; Y := \bar{0}_{q \times 1} ; \; z_j := 0, \; K_j := \bar{0}_{q \times 1} \]

2. Recording the events and the DES outputs.
   Wait until a new event \( e_{a_i} \) and the corresponding output vector \( y_{a_i} \) are observed. \( w := w e_{a_i} ; \; j := a_i \);
If $i > 1$ then goto 4.

3. Associating a transition to the first observed event.
   
   $z_j := z_j + 1$; $\beta_j := 1$; let $\hat{\alpha}$ and $t_{\hat{\alpha}}$ be such that
   
   $\epsilon_{\alpha} = \hat{\lambda}(t_{\hat{\alpha}})$; $t := t_{\hat{\alpha}}$; $Y(i, 1) := y_{a_1}$; $y = y_{a_1}$.
   
   goto 5;

4. Associating a transition to an event subsequent to the first one.
   
   4.1. Checking whether the event occurs for the first time.
   
   If $z_j = 0$ then
   
   $z_j := z_j + 1$; $\beta_j := 1$; let $\hat{\alpha}$ and $t_{\hat{\alpha}}$ be such that
   
   $\epsilon_{\alpha} = \hat{\lambda}(t_{\hat{\alpha}})$; $t := t_{\hat{\alpha}}$; $Y(i, 1) := y_{a_1}$; $y = y_{a_1}$.
   
   endif

4.2. Checking whether the event occurred previously.
   
   If $z_j > 0$ then
   
   if $j = \gamma$ then let $\theta := 2$
   
   else let $\theta := 1$
   
   endif

   flag=0;

   for all $h \in \{\theta, ..., z_j\}$
   
   if $K_j(h) = y_{a_1} - Y(i, i - 1)$ then
   
   $h := h$ and flag=1;
   
   endif

   endfor

4.2.1. Checking whether the transition to associate to the event is different from the ones previously considered.
   
   If flag=0 then
   
   $z_j := z_j + 1$; $K_j(z_j) := y_{a_1} - Y(i, i - 1)$; $\beta_j := 1$; let $\hat{\alpha}$ and $t_{\hat{\alpha}}$ be such that
   
   $\epsilon_{\alpha} = \hat{\lambda}(t_{\hat{\alpha}})$; $t := t_{\hat{\alpha}}$; $Y(i, 1) := y_{a_1}$; $y = y_{a_1}$.
   
   else (flag=1)
   
   $\beta_i := h$; $Y(i, 1) := y_{a_1}$; $t := t_{\hat{\alpha}}$

   endif

5. Updating the sequence and solving the ILP.
   
   $\sigma := \sigma'$; $\mu := 0$

   for all $j = 1, ..., \nu$

   if $z_j > 1$ then $\mu := \mu + z_j - 1$
   
   endif

   endfor

   $\hat{n} := \nu + \mu$; $\min \phi(\hat{\text{Pre}}, \hat{\text{Post}}, \hat{M}_\theta) \ s.t. \ \hat{\xi}(w, Y, \hat{\lambda}, \hat{n})$;

   $\hat{C} := \hat{\text{Post}} - \hat{\text{Pre}}$.

6. Updating the IPN incidence matrix.
   
   $l := 0$

   for all $r \in \{2, ..., z_j\}$

   if $K_r(r) = y_{a_1} - \hat{M}_\theta$, then $r := r$ and $l := 1$;

   endif

   endfor

   if $l = 1$ and $\hat{C}(r, 1) = \hat{C}(t_{\hat{\alpha}})$ then

   $\hat{C} := [\hat{C}(r, 1: l - 1), \hat{C}(r + 1: n)]$.

   endif

7. Returning to the condition of recording the events.
   
   Set $i := i + 1$ and goto 2.

B. Characterization of the Observed Event Sequence.

The application of the IA can provide different solutions depending on the observed word $w$. Hence, in this paper we investigate on the conditions that lead to determine an IPN that suitably models the DES.

Definition 6: Let us assume that $\langle Q, M_\theta \rangle$ with $Q = \{P, T, \text{Pre}, \text{Post}, E, Y, \Lambda, \Psi \}$ is a deterministic IPN system over $\lambda$ modeling the given DES such that $L(\langle Q, M_\theta \rangle) = L$. Moreover, let us consider the observed sequence $w = \lambda(\sigma)$ and the corresponding outputs $Y$. Denoting $\langle \hat{Q}, \hat{M}_\theta \rangle$ with $\hat{Q} = \{P, T, \hat{\text{Pre}}, \hat{\text{Post}}, E, Y, \Lambda, \Psi \}$ the IPN system obtained by observing $\sigma$ and applying the IA, the error of the identified IPN is defined as follows:

$$E_{\sigma} := |(\hat{M}_\theta - M_\theta)| + |(\hat{C} - C)\tilde{1}_{w=1}|, \quad (10)$$

where $|(.)|$ is a function that substitutes each element of its input with its absolute value.

$C = \text{Post} - \text{Pre}$, $\hat{C} := \hat{\text{Post}} - \hat{\text{Pre}}$.

Definition 7: Let us consider a DES and let $E$ be the DES event set. Moreover, let $w \in L$ be a sequence of events generated by the DES and $\langle Q, M_\theta \rangle$ an IPN modeling the system with $T$ the transition set of $Q$ and $\lambda$ its labeling function. We call the sequence $\sigma$, such that $w = \lambda(\sigma)$, transition complete if $\forall t \in T$ it holds $|\sigma|_j \geq 1$.

The following theorem characterizes the IPN system provided by the IA application.

Theorem 2: Let $\langle \hat{Q}, \hat{M}_\theta \rangle$ be an IPN system deterministic over $\lambda$ obtained by the IA after observing $w = e_{\alpha_1} e_{\alpha_2} ... e_{\alpha_k} = \lambda(\sigma)$ with $\sigma = t_{\alpha_1} t_{\alpha_2} ... t_{\alpha_k}$. Assuming $\Psi = L$ and $w \in \Psi^*$ with $q \leq m$, the IPN system denotes $E_{\sigma} := \begin{bmatrix} 0_{q=1} \\ \vdots \end{bmatrix}$ if the following conditions are verified:

2a) $\sigma$ is a transition complete sequence;

2b) $\forall e_{\alpha_i}, e_{\alpha_j} \in w$ with $e_{\alpha_i} = e_{\alpha_j}$ and $i \neq j$, $\sigma$ is a transition complete sequence.
\[ \forall t_{\beta_i}^{\alpha_i}, t_{\beta_j}^{\alpha_j} \in T^{e_{\alpha_i}} \text{ with } t_{\beta_i}^{\alpha_i} \neq t_{\beta_j}^{\alpha_j} \text{ it holds } y_{a_i} - y_{a_{i-1}} \neq y_{a_j} - y_{a_{j-1}}, \text{ where } y_{a_\theta} = M_{mK} \]

Proof:

Since by assumption it holds \( y_{a_\theta} = \Psi M_0 = M_{mK} \), we get:

\[ M_{pw} = \begin{bmatrix} M_{mK} \\ M_{mn} \end{bmatrix}. \]  \(\text{(11)}\)

Now, \( \forall e_{\alpha_i}, e_{\alpha_j} \in \omega \) with \( i \neq j \) two situations can occur:

a) \( e_{\alpha_i} \neq e_{\alpha_j} : \forall t_{\beta_i}^{\alpha_i} \in T^{e_{\alpha_i}} \text{ and } t_{\beta_j}^{\alpha_j} \in T^{e_{\alpha_j}} \text{ the IA sets } t_{\beta_i}^{\alpha_i} \neq t_{\beta_j}^{\alpha_j} \).

b) \( e_{\alpha_i} = e_{\alpha_j} : \) by 2b) the IA sets \( t_{\beta_i}^{\alpha_i} \neq t_{\beta_j}^{\alpha_j} \).

If \( \sigma \) is transition complete then \( \forall t \in T \), we can select in \( \sigma \) at least one transition \( t_{\beta_i}^{\alpha_i} \in \sigma \) such that \( t_{\beta_i}^{\alpha_i} = t \). Hence, we collect such transitions in the set \( T_\sigma \) defined as follows:

\[ T_\sigma = \{ t_{\beta_i}^{\alpha_i} : t_{\beta_i}^{\alpha_i} \in \sigma \text{ and } \forall t \in T \text{ one and only one transition } t_{\beta_i}^{\alpha_i} \text{ is chosen such that } t_{\beta_i}^{\alpha_i} = t \}. \]

Consequently, we obtain:

\[ \forall t_{\beta_i}^{\alpha_i} \in T_\sigma \text{ } C_{w} I_{\beta_i}^{\alpha_i} = (Post_{w} - Pre_{w}) I_{\beta_i}^{\alpha_i} = \begin{bmatrix} y_{a_i} - y_{a_{i-1}} \\ M_{mn} - M_{mn} \end{bmatrix}. \]  \(\text{(12)}\)

hence:

\[ \forall t_{\beta_i}^{\alpha_i} \in T_\sigma \text{ } C_{w} I_{\beta_i}^{\alpha_i} = y_{a_i} - y_{a_{i-1}} = M_{mK} - M_{mK} = C_{w} I_{\beta_i}^{\alpha_i}. \]  \(\text{(13)}\)

Then by (11), (13) and the definition of the error, we infer that

\[ E_{r_i} = \begin{bmatrix} 0 \\ \Psi_{\alpha_i} \end{bmatrix}. \]

IV. A CASE STUDY

A. The System Description

The case study is a three tank system (see Fig. 1) described in detail in [13]. The cylindrical tanks T1, T2 and T3 have equivalent cross section \( A \) and are connected serially. Each tank is connected by cylindrical pipes controlled by valves V4 and V5, each with the same cross section \( A \). Located at T2 is the outflow valve V6 that has a circular cross section \( A \). The outflowing liquid is collected in a tank (omitted in Fig. 1), which supplies the pumps 1, 2 and 3 controlled by the valves V1, V2 and V3. Symbol \( Q_i \) for \( i=1,2,3 \) denotes the i-th pump flow rate. \( \alpha \) measures the liquid level in tank Ti for \( i=1,2,3 \), respectively. Moreover, we call \( a_{\alpha_i} \) with \( i=1,2,3 \) the outflow coefficient of the i-th tank. The balance equations of the tanks are reported in [13]. In addition, variables \( V_4, V_5, V_6 \in \{0,1\} \) denote the state of valves 4, 5 and 6, respectively (0 stands for closed and 1 for open valve). We assume that the flow rates \( Q(t) \) for \( i=1,2,3 \) exhibit a sinusoidal variation with \( 2\pi/500 \text{ rad/s} \) pulse as in [13].

B. System Identification

We describe the behavior of the system managed by the opening and closing of valves as a DES in which twelve events can occur. Particularly, Table I reports the measurable events driving the system behavior. On the other hand, the system has a set of output sensors associated with a set of output symbols \( y_i \) with \( i=1,\ldots,15 \) describing the changing operative conditions of the tanks and of the valves. Table II shows in detail the relationship between sensor conditions and output symbols. The system is simulated in the Matlab Simulink environment and the events (i.e., the opening and closing of the valves) are governed by periodic quadratic functions. The identification algorithm is applied in two cases, on the basis of the knowledge of the output symbols provided by the simulation.

![Fig. 1. The layout of the three tank system.](image)

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<th>TABLE I. EVENT DEFINITION</th>
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<th>TABLE II. OUTPUT VECTOR DEFINITION</th>
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<td>( V_4=1(=0) )</td>
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<tr>
<td>( V_5=1(=0) )</td>
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<td>( V_6=1(=0) )</td>
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<tr>
<td>( h_{1}&lt;0.10 )</td>
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<tr>
<td>( h_{2}&lt;0.05 )</td>
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<td>( h_{3}&lt;0.20 )</td>
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<td>( h_{4}&lt;0.40 )</td>
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<td>( h_{5}&lt;0.20 )</td>
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Case I. To apply the IA the following data are specified: \( E=\{e_i : i=1,\ldots,12\} \) (see Table I) and \( q=15 \) (see Table II). In addition we assign a place to each output symbol so that \( P=\{p_i : i=1,\ldots,15\} \) and \( \Psi=I_{15} \). The simulation starts assuming that the initial liquid levels of T1, T2 and T3 are \( h_1=0.20=0.30=0 \) respectively and all the valves are closed. Hence, the initial marking is known and is as follows:
The IA is applied for a simulation time of 2000 s and in the case of 35 observed events the ILP solution performed by GLPK version 4.8 [5] gives a solution in 28.5 s on a PC with a 1.73 GHz processor and 1 GB of RAM memory. The identified IPN model is characterized by a set of 22 transitions and a labelling function defining the following sets:

\[ T^0 = \{ s_1 \}, \quad T^{e_1} = \{ s_2, s_3 \}, \quad T^{e_2} = \{ s_4 \}, \quad T^{e_3} = \{ s_7 \}, \quad T^{e_4} = \{ s_1 \}, \]

\[ T^{e_5} = \{ s_5 \}, \quad T^{e_6} = \{ s_6 \}. \]

Moreover, the estimated matrices \( \text{Pre} \) and \( \text{Post} \) are described by the IPN shown in Fig. 2. In addition, since the hypotheses of Theorem 2 hold and \( m=q=15 \), we get \( \Theta_{x=0} = 0_{5 \times 1} \). Furthermore, Fig. 2 shows that valve V2, represented by places \( p_2, p_8 \) and by transitions \( t_3^3 \) and \( t_4^1 \), is isolated from the other places and transitions. This IPN structure is obtained since the liquid levels \( h_i \) with \( i=1,2,3 \) are in the same condition (see Table II) every time that the valve V2 changes state.

**Case 2.** In the second case we assume that \( q=13 \), the set of places and of events are unchanged and \( \Psi = \begin{bmatrix} 1_{13} & 0_{13 \times 2} \end{bmatrix} \).

Starting from the initial marking (14), after a transition complete sequence of 35 events (in 2000 sec) the IA provides the IPN in Fig. 3. Since in such a case places \( p_{14} \) and \( p_{15} \) are not measurable by a identified IPN is now characterized by a set of 16 transitions. Moreover, the hypotheses of Theorem 2 hold and we can verify that \( \Theta_{x=0} = 0_{13 \times 1} \).

**V. CONCLUSION**

The paper recalls a recursive identification algorithm to estimate an IPN modeling a DES previously proposed by the authors. The identifier stores the observed sequence of DES events and the corresponding outputs that can provide a partial knowledge of the DES states. Assuming that the IPN is deterministic, i.e., each event occurrence from a given state yields only one new state even if two or more transitions can be labeled with the same symbol, an ILP problem is established to select an IPN exhibiting minimal dimensions. The paper investigates on the conditions that lead to determine an IPN modeling the DES dynamics without error and simulate and analyze a case study to enlighten the efficiency and the simplicity of the proposed method that may provide a solution even if the knowledge of the system is very poor.

Future research will deal with characterizing the DES language in relationship with the identified IPN language.

**REFERENCES**


