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Abstract—The paper proves some properties of a previously proposed identification algorithm that builds on line the Petri net model of Discrete Event Systems (DES). The procedure uses the real time observation of the DES events and the corresponding available output vectors that partially provide the place markings. The paper shows how the considered identification method allows us to define a supervisory controller via monitor places enforcing generalised mutual exclusion constraints. To show the efficiency of the proposed approach, a communication gateway case study is presented.

I. INTRODUCTION

SYSTEM identification deals with choosing mathematical models from a known model set to characterize the input-output behavior of an unknown system from finite data. Modern man made systems are modeled as Discrete Event Systems (DES) and the DES behavior can be described by a language that specifies all the admissible sequences of events that the DES is capable of “processing” or “generating” [4]. The problem of identifying a language and an automaton from finite data has been explored in [1, 12]. Moreover, paper [13] presents an algorithm for the construction of a free-labeled PN, i.e., a net where each transition is associated to a unique label, from the knowledge of a finite set of its firing sequences. An analogous problem is solved in [2] that proposes a method to synthesize a PN on the basis of a set of structural constraints. In two recent papers [3,10] the authors propose a linear algebraic approach for the identification of a PN from the knowledge of the DES language, i.e., the finite set of strings generated by the set of transitions. The approach solves the identification problem by Integer Linear Programming (ILP) that is applied to λ-free labeled nets [3], i.e., nets where two transitions may share the same event label but no transition is labeled with the empty string.

The identification problem of DES on the basis of an on line identification approach has been applied by Chung et al. [5] that propose an on line modeling refinement technique able to incrementally update the observed sample path. Moreover, Meda et al. [14] use an incremental synthesis approach to identify an interpreted PN (i.e., an extension of PN) modeling a DES on the basis of the partial knowledge of input and output symbols. An on line identification approach is proposed by Dotoli et al. [6,7] assuming that the DES evolution and the initial state are not perfectly known and only an upper bound of the cardinality of the place set is given. Moreover, an Identification Algorithm (IA) waits until a new event occurs, updates the transition set and the labeling function, defines and solves an ILP problem. Hence, the set of places, of transitions and the λ-free labeling function are recursively determined.

This paper proves some important properties of the PNs provided by the IA presented in [6,7]. It shows that even if it is possible to obtain an infinite number of PN systems modeling the DES, under suitable properties, each PN system provided by the IA is characterized by the same incidence matrix. Consequently, we prove that the proposed method can be used to specify a supervisory controller via monitor places enforcing generalized mutual exclusion constraints [9]. To this aim we consider a communication gateway case study that is modeled by an identified λ-free labeled PN and is controlled by monitors enforcing mutual exclusions constraints.

II. BACKGROUND ON PETRI NETS

A. Petri nets

A PN is a bipartite graph [15] described by the four-tuple \( PN=(P, T, Pre, Post) \) where \( P \) is a set of places with cardinality \( m \), \( T \) is a set of transitions with cardinality \( n \), \( Pre: P \times T \rightarrow \mathbb{N}^{m \times n} \) and \( Post: P \times T \rightarrow \mathbb{N}^{m \times n} \) are the pre- and post-incidence matrices respectively, which specify the arcs connecting places and transitions. More precisely, for each \( p \in P \) and \( t \in T \) element \( Pre(p,t) \) (\( Post(p,t) \)) is equal to a natural number indicating the arc multiplicity if an arc going from \( p \) to \( t \) (from \( t \) to \( p \)) exists, and it equals 0 otherwise. Note that \( \mathbb{N} \) is the set of non-negative integers. The incidence matrix of the net is defined as \( C=\text{Post}-\text{Pre} \). A marking is a mapping \( M: P \rightarrow \mathbb{N}^{m} \), assigning to each place of the net a nonnegative number of tokens. A PN system \( \{PN, M_{0}\} \) is a net PN with an initial marking \( M_{0} \).

Given a PN system \( \{PN, M_{0}\} \) and a set of places \( P^{c} \subseteq P \), we denote by \( M^{c} \) the restriction of \( M \) to \( P^{c} \). Here, we consider the set of places partitioned into two disjoint subsets: \( P^{s} \), the set of measurable places, whose marking \( M^{s} \) is known, and \( P^{ak} \), the set of non measurable places, whose marking \( M^{ak} \) is
unknown. Moreover, the restriction of the incidence matrix \( C \) to \( P^j \) (\( P^n \)) is denoted \( C^j \) (\( C^n \)).

A transition \( t_j \in T \) is enabled at a marking \( M \) if and only if (iff) for each \( p \in E(t_j) \) it holds:

\[
M(p) \geq \text{Pre}(p,t_j)
\]

and we write \( M(t_j) \) to denote that \( t_j \in T \) is enabled at marking \( M \). When fired, \( t_j \) produces a new marking \( M' \), denoted by \( M(t_j)M' \) that is computed by the PN state equation:

\[
M' = M + C \cdot t_j
\]

where \( t_j \) is the \( n \)-dimensional firing vector corresponding to the \( j \)-th canonical basis vector.

Let \( \sigma = t_{i_1}, t_{i_2}, \ldots, t_{i_h} \) be a sequence of transitions (or firing sequence) and let \( h \) be its length, given by the number of transitions that \( \sigma \) contains. Moreover, the notation \( M(\sigma)M' \) indicates that the sequence of enabled transitions \( \sigma \) may fire at \( M \) yielding \( M' \). We also denote \( \tilde{\sigma} : T \to \mathbb{N}^n \) the firing vector associated with a sequence \( \sigma \), i.e., \( \tilde{\sigma}(t) = v \) if transition \( t \) is enabled at \( \tilde{\sigma}(i) \geq 1 \). A marking \( \tilde{M} \) is said reachable from \( \langle PN, M_0 \rangle \) iff there exists a firing sequence \( \sigma \) such that \( M_0(\sigma)M \). The set of all markings reachable from \( M_0 \) is the definition set of \( \langle PN, M_0 \rangle \) and is denoted by \( R(PN, M_0) = \{ M : \exists \sigma : M_0(\sigma)M \} \).

B. \( \lambda \)-free labeled Petri nets

The following definition specifies and characterizes the labeled PN considered in this paper. 

Definition 1. A labeling function \( \lambda : T \to E \) assigns to each transition \( t \in T \) a symbol \( e \in E \). The labeling function is \( \lambda \)-free according to \([8,15]\), i.e., the same label \( e \in E \) may be associated to more than one transition while no transition may be labeled with the empty string \( \varepsilon \).

In particular, we denote by \( T^0 = \{ t \in T | \lambda(t) = e \} = \{ t_1, t_2, \ldots, t_{i_\varepsilon} \} \) the set of transitions associated with the same event \( e \), and \( z_{i_\varepsilon} = \text{Card}(T^0) \), where \( \text{Card}(.) \) stands for the cardinality of the set (\( . \)).

Given a PN system \( \langle PN, M_0 \rangle \), the set of its firing sequences is denoted \( \mathcal{A}(PN, M_0) = \{ \sigma \in T^h | M_0(\sigma) \} \). Moreover, we denote as \( w \) the word of events associated with the sequence \( \sigma \) such that \( w = \lambda(\sigma) \), using the extended form of the transition labeling function \( \lambda : T^* \to E^* \) in the usual manner. In addition, the empty string \( \varepsilon \) is associated with the word of null length. Furthermore, we define the \( \lambda \)-free labeled language as the set of admissible words in \( E^* \) given the initial marking \( M_0 \), namely:

\[
\mathcal{A}(PN, M_0) = \{ w \in E^* | \lambda(\sigma) = w, \sigma \in T^*, M_0(\sigma) \}.
\]

III. DES IDENTIFICATION BY \( \lambda \)-FREE LABELED PETRI NETS

In this section we recall the identification problem definition and the results presented in \([6,7]\).

Let us consider a DES with event set \( E \) and language \( \mathcal{X} \). The inputs of the identifier are the DES event sequences \( w = e_\alpha e_{\alpha_2} \ldots e_{\alpha_h} \) with \( h \geq 1 \) and \( e_\alpha \in E \) for \( i = 1, \ldots, h \) and the corresponding output vectors \( y_0, y_1, \ldots, y_n \), where \( y_0 \) is the initial output vector and each \( y_i \in \mathbb{N}^q \) is the output vector observed after event \( e_{\alpha_i} \) with \( i = 1, \ldots, h \).

In order to define the problem of identifying a PN system \( \langle PN, M_0 \rangle \) modeling the DES, the following properties of the DES are assumed:

(A1) all the events of set \( E \) are observable, i.e., the events can be detected and distinguished;

(A2) the DES state can be (partially) observed by the measured output vectors \( y \in \mathbb{N}^q \);

(A3) the DES can be modeled by a PN system with \( \lambda \)-free labeling function \( \lambda \).

Moreover, we assume that the PN \( PN=\langle P, T, Pre, Post \rangle \) modeling the DES enjoys the following properties:

(A4) the subset of measurable places is \( P^h \subseteq P \) with \( \text{Card}(P^h) = q \);

(A5) an upper bound \( \bar{m} \geq q \) of \( \text{Card}(P) \) is given.

Now, we define the PN set \( D = \{ PN=(P,T,Pre,Post) : Pre \in \mathbb{N}^{m_h} \cdot Post \in \mathbb{N}^{m_h} \} \) so that the identification problem is formally stated as follows.

Identification Problem. Let us consider a DES with event set \( E \) and language \( \mathcal{X} \) verifying assumptions A1, A2 and A3. Let us observe an event sequence \( w \in \mathcal{X} \) and the corresponding output vectors \( y \in \mathbb{N}^q \). The Identification Problem consists in determining a place set \( P \) and its cardinality \( m \), a transition set \( T \) and its cardinality \( n \), a \( \lambda \)-free labeling function \( \lambda \) and a PN system \( \langle PN, M_0 \rangle \) satisfying assumptions A4 and A5 such that \( PN \in D, M_0 \in \mathbb{N}^m \) and \( w \in \mathcal{A}(PN, M_0) \).

To characterize the PN modeling the DES, the following definition is introduced.

Definition 2. Let us consider a DES modeled by the PN system \( \langle PN, M_0 \rangle \) with \( \lambda \)-free labeling function \( \lambda \) and \( PN=(P,T,Pre,Post) \). The PN system \( \langle PN, M_0 \rangle \) is said event detectable by the output iff \( w \in \mathcal{A}(PN, M_0) \) and \( \exists e_\alpha, e_{\alpha_2} \in E^* : e_\alpha = e_{\alpha_2} \) with \( i \neq j \), \( e_\alpha = \lambda(\hat{e_\alpha}) \), \( e_{\alpha_2} = \lambda(\hat{e_{\alpha_2}}) \) and \( t_{\alpha_i} \neq t_{\alpha_j} \) it holds \( y_i \neq y_j \). In other words, the PN system is event detectable iff any two transitions in \( T \) can be distinguished from each other by the observation of the DES output vectors.

A solution of the Identification Problem is obtained by the Identification Algorithm (IA) of Fig.1 described in detail in [7]. The IA waits until an event occurs and stores the corresponding output vector. In particular, the inputs of the IA are an upper bound \( \bar{m} \) of the number of places, the dimension \( q \) of the output vectors, the initial output \( y_0 \), the
events generated by the DES and the corresponding output vectors. Let us assume that the IA observes the DES event sequence \( w = e_{a_1}e_{a_2}...e_{a_h} \) with \( h \geq 1 \) and the corresponding output vectors \( y_0, y_1, ..., y_h \) that are stored in the \( q \times (h+1) \) matrix \( Y \) with \( Y(i, j+1) = y_j \) for \( i = 0, 1, ..., h \). At each observation the algorithm properly updates the labeling function \( \lambda \), the transition set \( T \) and the place set \( P \). Hence, the IA infers
\[
\sigma = \{ \sigma_1^1, \sigma_2^1, ..., \sigma_h^1 \} \quad \text{with} \quad \lambda(\sigma_i^1) = e_{a_i} \quad \text{and} \quad \lambda(\sigma_i^2) \in T \quad \text{for} \quad i = 1, ..., h.
\]
Denoting \( M = \text{Card}(P) \) and \( M_0 = \{ M_1^0, M_2^0, ..., M_h^0 \} \), the evolution of the net corresponding to the sequence \( \sigma \), we assume that \( M_i^k \) is the restriction of the marking \( M_i \) to \( P^k \) and \( M_i^{nk} \) is the restriction to \( P^{nk} \). Now, the following linear algebraic constraint set is defined:

\[
\xi(w, Y, \lambda, T, m) = \begin{cases} 
\text{Pre. Post } \in \mathbb{N}^{m \times n}, \\
M_i \in \mathbb{N}^m \quad \text{with} \quad i = 0, ..., h \\
\text{Post } \hat{1}_{ncd} + \text{Pre } \hat{1}_{ncd} \geq \hat{1}_{ncd} \\
\forall h_i^0 \in \sigma : \lambda(\sigma_i^1) = w \text{ Pre } \hat{1}_{h_i^0} \leq M_i \\
\forall h_i^0 \in \sigma : \lambda(\sigma_i^1) = w \text{ Post } \hat{1}_{h_i^0} = M_i - M_{i-1}
\end{cases}
\]

where \( \hat{1}_{m \times n} \) is the matrix of dimensions \( m \times n \) with each element being 1.

As an indicator of the size of the PN, we introduce a linear function \( \phi : \mathbb{D} \times \mathbb{N}^m \to \mathbb{N} \) defined by a linear combination of matrices \( \text{Pre} \) and \( \text{Post} \) and of the initial marking \( M_0 \):

\[
\phi(\text{Pre, Post, } M_0) = \hat{a}^\top \text{Pre } \hat{b} + \hat{c}^\top \text{Post } \hat{d} + \hat{e}^\top M_0,
\]

where the vectors \( \hat{a}, \hat{c}, \hat{e} \in \mathbb{N}^m \) and \( \hat{b}, \hat{d} \in \mathbb{N}^n \) are appropriately chosen and at least one of them is non-zero. Once \( P, T \) and \( \lambda \) are determined, it is shown in [6,7] that a solution of the Identification Problem is a solution of the following ILP problem:

\[
\text{min } \phi(\text{Pre, Post, } M_0) \text{ s.t. } \xi(w, Y, \lambda, T, m).
\]

At each event occurrence the IA redefines the ILP problem and provides the output named Solution: it holds Solution = (\( \{ \text{PN}_v, M_{0n} \}, \lambda_{0v} \)) if the IA provides a PN system with labeling function \( \lambda_{0v} \), \( \text{PN}_v(\{ P_v, T_v, \text{Pre}_v, \text{Post}_v \}) \) and initial marking \( M_{0n} \), whereas Solution = 0 if the defined ILP problem does not admit any solution. In [7] we proved that the solution provided by the IA is a solution of the Identification Problem.

We remark that the proposed IA may be applied in real time storing the DES evolution (steps 1-4) and subsequently solving at least an ILP problem at each event occurrence. Obviously, in order to apply the IA online, the dynamics of the DES has to be slow with respect to the time required to solve the ILP problem at each occurrence.

Identification Algorithm (IA)

1. Input: \( q, m, y_0 \)
2. Output: Solution = (\( \{ \text{PN}_v, M_{0n} \}, \lambda_{0v} \)) or Solution = 0
3. Initializing the algorithm variables.
4. Recording the events and the DES outputs.
5. Solving the ILP problem.
6. Returning to the condition of recording the events.

IV. AN APPLICATION TO THE SUPERVISORY CONTROL SPECIFICATION

In this section we characterize the PN system obtained by the IA and we show how the proposed identification method
can be applied to synthesize a supervisor control of the PN systems via monitor places.

A. Characterization of the identified PN system

Let us consider a DES with event set $E$ and language $\mathcal{I}$ satisfying assumptions A1-A3 and let $\langle PN, M_0 \rangle$ be a PN with labeling function $\lambda$ modeling the DES such that $PN=(P,T,Pre,Post)$. Assume that the sequence $w=e_1t_{i_1}e_2t_{i_2}...e_3t_{i_3} \in \mathcal{I}$ and the output vectors $y_i \in \mathbb{N}^q$ for $i=0,...,h$ are observed. Let $\langle PN_w, M_{0w} \rangle$ with labeling function $\lambda_w$ and $PN_w=(P_{w},T_{w},Pre_{w},Post_{w})$ be a PN system identified by the IA. Denoting by $C_w^k$ the restriction of matrix $C_w$ to $P_{w}^k \subseteq P_{w}$, the following proposition characterizes the PN system provided by the IA application.

Proposition 1. If the PN system modeling the DES is event detectable by the output vector $y_i \in \mathbb{N}^q$ and sequence $w$ such that $w=w(\sigma)$ is transition complete, then the following conditions hold for any PN system $\langle PN_w, M_{0w} \rangle$ obtained by the IA: P1a) $T_{w}=T$; P1b) $C_w^k = C^k$.

Proof. By the application of the IA, $\forall e_{i_k}, e_{i_j} \in w$ with $i\neq j$ three situations can occur: a) $e_{i_k} \neq e_{i_j}$: the IA sets $\lambda(t_{i_k}^{a_j}) = e_{i_j}$, $\lambda(t_{i_j}^{a_j}) = e_{i_k}$ and $t_{i_k}^{a_j} \neq t_{i_j}^{a_j}$, hence $t_{i_k}^{a_k} \in T_w$; b) $e_{i_k} = e_{i_j}$ and $y_jy_{\cdot}y_{\cdot}y_{\cdot}y_{\cdot}$: the IA sets $\lambda(t_{i_k}^{a_j}) = e_{i_k}$, $t_{i_k}^{a_j} \neq t_{i_j}^{a_j}$ and $t_{i_k}^{a_k} \in T_w$; c) $e_{i_k} = e_{i_j}$ and $y_jy_{\cdot}y_{\cdot}y_{\cdot}y_{\cdot}$: the IA sets $\lambda(t_{i_k}^{a_j}) = \lambda(t_{i_j}^{a_j}) = e_{i_k}$, $t_{i_k}^{a_k} \neq t_{i_j}^{a_j}$ and $t_{i_k}^{a_k} \in T_w$. If $\sigma$ is transition complete and the PN system $\langle PN, M_0 \rangle$ is event detectable by the output, then by construction $\forall t \in T$ there exists one and only one transition $t_{i_k}^{a_k} \in T_w$ such that $t_{i_k}^{a_k}=t_j$. Consequently, we infer $T_{w}=T$ and P1a) is true. Since $P_{w}^{k} \subseteq P_{w}$, the following $n$ equations are determined by the state equation (2): $\forall t_{i_k}^{a_k} \in T_w$:

$$C^k_{i_k} t_{i_k}^{a_k} = y_{t_j} - y_{t_{i_k}} = M^k_{i_k} - M^{k+1}_{i_k} = C^k t_{i_k}^{a_k}.$$  

Consequently we prove that $C^k_w = C^k$.

Proposition 1 points out that if the PN system is event detectable by the output symbols and if $\sigma$ such that $w=w(\sigma)$ is transition complete, then there exists only one matrix $C^k_w = C^k$ obtained by the IA. Indeed, each identified PN system is characterized by the same incidence sub matrix restricted to $P^k$. Obviously, if $P^k = P^w = P$ and the hypotheses of Proposition 1 hold, then the identified PN system denotes $C_w = C$.

B. The supervisory control specification

We recall the problem of enforcing Generalized Mutual Exclusion Constraints (GMECs) on PN [9]. Let $\langle PN, M_0 \rangle$ be a PN system, whose set of reachable markings is $R(PN, M_0)$. Assume that we are given a set of legal markings $G \subseteq \mathbb{N}^m$ and consider the basic control problem of designing a supervisor that restricts the reachability set of the plant in closed loop to $R_{G}:=R(PN, M_0)$. We denote a GMEC as a couple $(l, H)$ where $l: P \rightarrow \mathbb{Z}$ is a $1 \times m$ weight vector, $H \in \mathbb{Z}$ and $H$ is the set of relative numbers. A set of GMECs $(L, H)$, with $L=[l_1^T l_2^T \ldots l_n^T]^T$ and $H=[H_1 H_2 \ldots H_n]^T$, defines the legal marking set $\mathcal{M}(L,H)$.

Proof. The proof is straightforward and can be omitted. By Proposition 2, the hypothesis that the PN system is identified by a transition complete sequence guarantees that the set of GMECs imposed on the measurable places are enforced by a controller that is independent from the observed event sequence.

V. THE CASE STUDY

A. System Description

In this section we address the problem of identifying the
architecture of a communication gateway between two kinds of networks. More precisely, the Call Control Application (CCA) has a sending and a receiving interface that are performed by two totally independent processes. Moreover, a service handler interface manages the specific end user service transfer through the gateway (voice, data, messaging). We represent the behavior of the system as a DES whose events are shown in Table I. A set of output sensors associated with a set of output symbols describes the changing DES operative conditions as depicted in Table II. In particular, the sensors may provide the number of idle users, calls or occupied positions in the communication buffer between sending interface and receiving interface.

<table>
<thead>
<tr>
<th>TABLE I.</th>
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<tbody>
<tr>
<td>EVENT DEFINITION.</td>
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<tr>
<td>e₁ Connect indication</td>
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<tr>
<td>e₂ Dispatching to buffer</td>
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<tr>
<td>e₃ Connect request</td>
</tr>
<tr>
<td>e₄ Connect response</td>
</tr>
<tr>
<td>e₅ Connect confirm</td>
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<tr>
<td>e₆ Alert request</td>
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<tr>
<th>TABLE II.</th>
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<tr>
<td>OUTPUT VECTOR DESCRIPTION.</td>
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<td>Description</td>
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<tr>
<td>Call active</td>
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<td>Call received</td>
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<tr>
<td>Call delivered</td>
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<tr>
<td>Occupied positions in buffer</td>
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<tr>
<td>Call incoming</td>
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<tr>
<td>Call present</td>
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<td>Idle user</td>
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<th>TABLE III.</th>
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<tr>
<td>OBSERVED OUTPUT VECTORS.</td>
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<td>y₂</td>
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<td>y₁₇</td>
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<td>y₃</td>
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<td>y₂₉</td>
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<td>y₁₅</td>
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<td>y₃₀</td>
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With reference to the DES identification methodology recalled in Section III, we suppose that assumptions A₁-A₅ are satisfied. We identify the system in two cases: in the first case (Case A) we assume that the complete system state is known by the output, i.e., q=ₕₐₗₐₜ₆=ₗ₉; in the second case (Case B) we assume that just a partial observation of the system is allowed, i.e., q=₅.

B. System Identification: Case A

The IA observes the following sequence w:

\[ w = e_{α₁} \ldots e_{αₐₜ} = \]

\[ e₁ e₂ e₃ e₄ e₅ e₆ e₇ e₈ e₉ e₁₀ e₁₁ e₁₂ e₁₃ e₁₄ e₁₅ \]

Moreover, assuming q=₇, the observed outputs are reported in Table III. In addition, we suppose that the DES evolution starts with 4 idle users in the communication gateway, i.e., the initial output is \( yₐₜ=[0 0 0 0 0 0 4]T \). The objective function is the following

\[ φ(Pre, Post, M₀) = \tilde{I}_{ₐₜ₇ₐ₉₃}(Pre + Post)I_{ₐₜ₇ₐ₉₃} + \tilde{I}_{ₐₜ₇ₐ₉₃}M₀ \],

and some constraints are added to the constraint set (4). In particular, we impose that each place is connected with an input transition and an output transition. Hence the following constraints have to be verified:

\[ Pre \cdot I_{ₐₕₐₜ₆} ≥ I_{ₐₜ₇ₐ₉₃} \] and \[ Post \cdot I_{ₐₜ₇ₐ₉₃} ≥ I_{ₐₜ₇ₐ₉₃} \]

The IA is applied and the ILP problem (6) is solved using the GLPK software version 4.8 [11]. However, the IA provides no solution until the occurrence of the event \( e_{α₇} = e₇ \). After the observation of the entire word w, the ILP provides a solution in an average time of 0.41 s with a PC equipped with a 1.73 GHz processor and 1 GB RAM. The identified PN system is shown in Fig. 2. In particular, the obtained transition set \( Tₖ \) contains 19 transitions and the labeling function \( λ_{ₚₜ} \) defines the following sets:

\[ Tⁿ = \{ t₁ \}, \quad Tⁿ² = \{ t²₁, t²₂ \}, \quad Tⁿ₅ = \{ t₅₁ \}, \quad Tⁿ₄ = \{ t₄₁, t₄₂ \}, \]

\[ Tⁿ₃ = \{ t₃₁, t₃₂ \}, \quad Tⁿ₆ = \{ t₆₁ \}, \quad Tⁿ₇ = \{ t₇₁ \}, \quad Tⁿ₉ = \{ t₉₁ \}, \]

\[ Tⁿ₉ = \{ t₉₁, t₉₂, t₉₃, t₉₄, t₉₅, t₉₆ \}, \quad Tⁿ₁₀ = \{ t₁₀₁ \}, \quad Tⁿ₁₁ = \{ t₁₁₁ \}. \]

Assuming that the PN system modeling the DES is event detectable by the output and the sequence σ obtained by the labeling function \( λ_{ₚₜ} \) is transition complete, since the output provides the complete marking of the place set \( (q=₉₉ₖₖ₆=ₗ₉) \), by Proposition 1 we infer that the incidence matrix of the identified PN is such that \( Cₕₐₜ₆₉₉₆=ₗ₉ \)

C. System Identification: Case B

In this case we assume that it is not possible to detect the number of idle users and whether a call is present in the network, i.e., \( yₚ \in N^{₉₉₆=ₗ₉} \). More precisely, the identifier observes the event sequence (7) and the corresponding output symbols, represented by the first 5 elements of the output vectors shown in Table III. The identification procedure defines the ILP (6) with the same objective function and the
same constraint set of Case A. Starting from the initial output vector $y_0=[0 0 0 0 0]^T$, the IA is applied to identify the PN modeling the CCA assuming that $m=q=5$ and observing the event sequence (7). In this case the procedure provides the PN shown in Fig. 3 (solid lines only) using GLPK software in an average time of 0.47s. We enlighten that the set of transitions is the same set obtained in case A, where $10000$ incidence matrix obtained in Case A.

Since the hypotheses of Proposition 2 are satisfied and the set of GMECs is imposed on the measurable places, the incidence matrix and the initial marking of the monitor places enforcing the GMECs are the same for Case A and for Case B.

VI. CONCLUSION

The paper analyzes the properties of a DES real time identification procedure, previously proposed by the authors. The considered technique synthesizes on line a $A$-free labeled PN modeling the observed deterministic DES behavior on the basis of partial knowledge of the system state and the output symbols. The paper shows how the identification method may be applied for defining a supervisory controller via monitor places enforcing a set of generalized mutual exclusion constraints. To show the efficiency of the proposed approach, a communication gateway case study proposed in the related literature is considered.

Future research deals with identifying PN models of non deterministic DES that may be driven also by silent events.

REFERENCES