An Urban Traffic Network Model by First Order Hybrid Petri Nets

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Abstract—The paper proposes a model for real time control of urban traffic networks. A modular framework based on first order hybrid Petri nets models the vehicle flows by a first order fluid approximation. Moreover, the lane interruptions and the signal timing plan controlling the area are described by the discrete event dynamics using timed Petri nets. The proposed model is applied to a real intersection located in Bari, Italy. Simulation of different scenarios shows the technique efficiency; validation is performed by comparison with a previously proposed alternative approach employing colored Petri nets.

Keywords—Hybrid Petri nets, modeling, urban traffic control, simulation.

I. INTRODUCTION

Two main approaches are developed in the literature to model and analyze the behavior of vehicles in Traffic Networks (TN) and may be classified according to the level of modeling detail. First, microscopic models describe both the system entities and their interactions with a high level of detail. However, these high-fidelity microscopic models and the resulting software are costly to develop and execute. Second, models represent traffic streams in an aggregate manner, e.g. by scalar values of flow rate, density and speed. These lower fidelity models are less costly to develop and maintain, but their representation may be inaccurate.

Petri Net (PN) models describe effectively signal traffic control and concurrent activities in TN [3, 8]. Di Febbraro, et al. [4, 5] present a traffic model in a timed PN framework: tokens are vehicles and places are parts of lanes and intersections. However, in ordinary PN tokens can not distinguish among different vehicles and their routes. Colors are introduced in [3, 6]: the token color of a Colored Timed Petri Net (CTPN) represents the path a vehicle has to follow and places are cells accommodating one vehicle. The CTPN model in [6] generates the code to simulate intersections and test the actuated control strategy proposed in [7].

The mentioned models share the limitation that vehicles are modeled by discrete quantities (tokens), a not realistic assumption in large systems: the state space of the TN model is excessively large, so that incompatibilities in the simulation and performance optimization often arise, leading to large computational efforts. Conversely, PN formalisms using fluid approximations provide an aggregate formulation to deal with complex systems, reducing the dimension of the state space.

Di Febbraro et al. [5] propose Hybrid Petri Nets (HPN) to model an urban TN. Modeling the TN as a hybrid system by means of HPN allows taking advantage of modeling traffic flows as fluids and traffic lights as event-driven dynamics. Even if the model is efficient and able to estimate the network performance, it needs numerous input parameters that have to be partially measured or estimated. Some of these parameters require an identification algorithm based on real traffic data.

This paper proposes an effective and efficient modular model for traffic management and control based on First Order Hybrid Petri Nets (FOHPN) [1] that are a HPN formalism including continuous places holding fluid, discrete places containing a non-negative integer number of tokens and transitions, which are either discrete or continuous. FOHPN were selected for traffic modeling since they present several key features. First, their use typically leads to a considerable increase in computational efficiency with respect to place/transition models, since the simulation of fluid models can often be performed much more efficiently. Second, fluid approximations provide an aggregate formulation to deal with large TN, thus reducing the dimension of the state space. The proposed model is built by using a modular approach based on the idea of the bottom-up methodology. In particular, vehicle flows are described by the continuous dynamics. Moreover, random lane interruptions and traffic lights are described by the discrete event dynamics. The effectiveness of the FOHPN formalism is shown by applying the technique to an intersection of the urban area of Bari, Italy, whose dynamics is simulated in different traffic scenarios in the MATLAB environment and compared with the CTPN model in [6].

II. BASICS OF FIRST ORDER HYBRID PETRI NETS

A. The FOHPN structure and marking

A FOHPN is a bipartite digraph described by the six-tuple $PN=(P, T, Pre, Post, \Delta, F)$. The set of places $P=P_d\cup P_c$ is partitioned into a set of discrete places $P_d$ (represented by circles) and a set of continuous places (represented by double circles). The set of transitions $T=T_d\cup T_c$, is partitioned into a set of discrete transitions $T_d$ and a set of continuous transitions $T_c$ (represented by double boxes). The set of discrete transitions $T_d=T_d\cup T_d\cup T_d$ is further partitioned into a set of immediate transitions $T_i$ (represented by bars), a set of stochastic transitions $T_s$ (represented by boxes) and a set of deterministic timed transitions $T_d$ (represented by black boxes). We denote
$T_{f}=T_{d}\cup T_{p}$, indicating the set of timed transitions.

Matrices $Pre$ and $Post$ are respectively the $|P|\times|T|$ pre- and post-incidence matrix, where $|A|$ denotes the cardinality of set $A$. Such matrices specify the net digraph arcs and are defined as follows: $Pre, Post : \begin{bmatrix} P_{c} \times T \rightarrow \mathbb{R}^{+} \end{bmatrix}$. We require that $\forall t \in T_{c}$ and $\forall p \in P_{c} \mathrm{Pre}(p,t) = \mathrm{Post}(p,t)$ (well-formed nets).

Function $\Delta : T_{c} \rightarrow \mathbb{R}^{+}$ specifies the timing of timed transitions. In particular, each $t_{j} \in T_{c}$ is associated the average firing delay $\Delta(t_{j})=1/\lambda_{j}$, where $\lambda_{j}$ is the average transition firing rate. Each $t_{j} \in T_{D}$ is associated the constant firing delay $\Delta(t_{j})=FT_{j}$. $F: T_{c} \rightarrow \mathbb{R}^{+} \times \mathbb{R}^{+}$ specifies the firing speeds of continuous transitions, where $\mathbb{R}^{+} = \mathbb{R} \setminus \{0\}$. For any $t_{j} \in T_{c}$, we let $F(t_{j})=(V_{m_{j}},V_{M_{j}})$, with $V_{m_{j}} \leq V_{M_{j}}$, where $V_{m_{j}} (V_{M_{j}})$ is the minimum (maximum) firing speed of the transition.

Given a FOHPN and a transition $t \in T$, we define $\bullet \{ p \in P : \mathrm{Pre}(p,t)>0 \}$ and $\bullet \{ p \in P : \mathrm{Pre}(p,t)>0 \}$ (pre- and post-set of $t$, respectively). The corresponding restrictions to continuous or discrete places are $\{ p \in P _{d} : \mathrm{Pre}(p,t)>0 \}$ of $\{ p \in P _{d} : \mathrm{Pre}(p,t)>0 \}$. Similar notations may be used for pre-and post-sets of places. The net incidence matrix is $C(p,t)=\mathrm{Post}(p,t)-\mathrm{Pre}(p,t)$. The restriction of $C$ to $P_{X}$ and $T_{X}$ (with $X \in \{ c,d \}$) is $C_{XY}$. A marking $m : \begin{bmatrix} P_{d} \rightarrow \mathbb{N} \end{bmatrix}$, $\begin{bmatrix} P_{c} \rightarrow \mathbb{R}^{+} \end{bmatrix}$ is a function assigning each discrete place a non-negative number of tokens (represented by black dots) and each continuous place a fluid volume; $m_{i}$ denotes the marking of place $p_{i}$. The value of a marking at time $t$ is $m(t)$.

The restrictions of $m$ to $P_{d}$ and $P_{c}$ are $m^{d}$ and $m^{c}$. A FOHPN system $\langle P,N,m(t_{0}) \rangle$ is a FOHPN with initial marking $m(t_{0})$. Continuous and discrete transitions fire as follows: 1) a discrete transition $t_{j} \in T_{d}$ is enabled at $m$ if for all $p_{i} \in \bullet \{ t_{j} \} m_{i} \geq \mathrm{Pre}(p_{i},t_{j})$, if $t_{j} \not\in T_{D}$; 2) a continuous transition $t_{j} \in T_{c}$ is enabled at $m$ if for all $p_{i} \in \bullet \{ t_{j} \} m_{i} \geq \mathrm{Pre}(p_{i},t_{j})$. Moreover, an enabled transition $t_{j} \in T_{c}$ is said strongly enabled at $m$ if for all $p_{i} \in \bullet \{ t_{j} \} m_{i} > \mathrm{Pre}(p_{i},t_{j})$. The transition firing speeds of a continuous transition $t_{j}$, $\Delta(t_{j})=F(t_{j})$, is constant.

The net evolution at the occurrence of a macro-event is:

$$m^{c}(\tau_{k}) = m^{c}(\tau_{k}) + C_{cc} \nu(\tau_{k})(\tau_{k} - \tau_{k})$$

$$m^{d}(\tau_{k}) = m^{d}(\tau_{k})$$

The firing count vector of a discrete transition $t_{j}$ at $\tau_{k}$ is $\nu(\tau_{k})$.

We associate to each $t_{j} \in T_{c}$ a timer $\nu(\tau_{k})$ and $\sigma(t_{j})$. The execution of a timed transition $t_{j}$ at $\tau_{k}$ is $\nu(\tau_{k})$. Hence, the timer evolution within period $[\tau_{k},\tau_{k+1})$ is $\nu(\tau_{k})=\nu(\tau_{k})$ if $t_{j}$ is not enabled; $\nu(\tau_{k})=\nu(\tau_{k})+\nu(\tau_{k})$ if $t_{j}$ is enabled. When $t_{j}$ is disabled or fires, its timer is reset to zero.

The state space at time $\tau_{k}$ is denoted by the marking $m(\tau_{k})$, $\nu(\tau_{k})$. The system state at time $\tau_{k}$, given by the marking and timers, is $\chi(\tau_{k})=[m^{c}(\tau_{k}) \ m^{d}(\tau_{k}) \ \nu(\tau_{k})]^{T}$. The system input is $\mathbf{u}(\tau_{k})=[\nu(\tau_{k}) \ \sigma(t_{j})]^{T}$, collecting the current macro-period length and the transition that will fire at the end of such macro-period. A FOHPN system can be described in $[\tau_{k},\tau_{k+1})$ by a linear discrete-time time-varying state variable model [2]:

$$\chi(\tau_{k+1}) = A(\tau_{k}) \chi(\tau_{k}) + B(\tau_{k}) \mathbf{u}(\tau_{k})$$

III. MODELLING A SIGNALIZED INTERSECTION AREA

In the proposed TN model, the fundamental components are signalized intersections, links and vehicles. Each link represents the space available between two adjacent intersections and includes one or several lanes. Hence, a generic signalized TN comprises a number of junctions controlled by traffic lights pertaining to a common semaphoric cycle, including a set $L=[L_{k}]_{k=1\ldots K}$ of $K$ links. Link $L_{k}$ of length $l_{k}$ with $k \in \{1\ldots K\}$ includes one ore more lanes of finite capacity $C_{p}>0$ each, denoting the maximum vehicle number (Passenger Car Units, PCUs) a lane can simultaneously accommodate. Figure 1 shows a junction composed of $K=14$ links: $L_{1}$, $L_{4}$, $L_{7}$ and $L_{8}$ are composed by two lanes, the remaining ones include one lane only. We assume that lane changes are allowed in road. Moreover, it is necessary to take
into account the physical space a vehicle crossing the intersection occupies. Such a space, named intersection cell, can be occupied by one or more vehicles and may coincide with the whole intersection area in a very simple intersection or with a part of the physical space in a multi-lane intersection.

A. The urban area model

Based on the bottom-up approach, we propose a modular FOHPN model to describe a TN. More precisely, the TN can be divided into the following entities modeled by FOHPN modules: lane cells and intersection cells. These sub-models are then interconnected to form the model of the whole TN.

The continuous dynamics models the flow of vehicles in the system: lane and intersection cells are described by continuous places and vehicles are continuous flows (fluids). We consider discrete events occurring stochastically in the system, e.g., the blocking of a lane due to unpredictable events. Hence, the state of the TN at the beginning of each macro-period is a vector $x(t_0)$ including the following sub-vectors: $m'(t_0)$, collecting the markings of the continuous places, i.e., a cell in a lane or in a crossing section; $m'(t_0)$, collecting the markings of the discrete places, i.e., the places modelling a lane block or a traffic light; the timers vector $v(t_0)$, collecting the values of the timers of discrete timed transitions.

The following FOHPN modules model the TN subsystems.

B. The lane cell model

We assume that each lane is divided into cells of limited capacity according to the lane length. Figure 2 depicts the model of two cells $i$ and $j$ pertaining to two adjacent lanes of the same link. Each cell is modeled by two continuous places ($p_i$ and $p'_j$, model cell $i$, $p_j$ and $p'_j$, model cell $j$). In particular, consider cell $i$ in Fig. 2: marking $m_i$ represents the vehicles in the cell, while $m'_j$ describes the available space. Hence, if cell $i$ can accommodate $C_i$ vehicles and is initially empty, the initial markings are $m_i(0)=0$ and $m'_j(0)=C_i$, as shown in Fig. 2.

With reference to Fig. 2, vehicles that enter (exit from) cell $i$ are modeled by transition $t_i$ ($t_{0i}$). We assign the firing speeds $F(t_{0i})=F(t_i)=V_{mi}$, $V_{mi}=0$, $V_{mi}=V_i$ represents the minimum (maximum) vehicle speed in the lane. Weight $r_i$ in Fig. 2 represents the fraction of vehicle flow remaining in the lane and $(1-r_i)$ is the changing lane fraction. The traffic flow can be interrupted due to unpredictable events. These situations are represented by the FOHPN of Fig. 3, where $t_i$ models an input or an output transition of a cell.

If the discrete place $p_{down}$ is marked, continuous transition $t_i$ is enabled and vehicles flow regularly. On the contrary, if $t_i$ fires, $p_{down}$ is marked and the lane becomes blocked, i.e., $t_i$ is not enabled. When $t_i$ fires, the regular travel of vehicles is restored. Typically, we assume $FT_{p_i}<<FT_{p_{down}}$.

C. The intersection cell model

The intersection crossing area is divided into intersection cells, each modeled by the FOHPN in Fig. 4. An intersection can be occupied by vehicles that follow different paths. Hence, each vehicle path crossing an intersection cell is described by a place but all the involved places share the same capacity place. For example, Fig. 4 shows an intersection cell where two paths are allowed (paths $a$ and $b$) and continuous places $p_1$ and $p_0$ accommodate vehicles that follow path $a$ and $b$, respectively. Moreover, continuous place $p_c$ represents the available space of finite capacity $C_c$ of the intersection cell. Hence, $m_i(t)=C_i m_1(t)+m_2(t)$ $\forall t \in \mathbb{R}^+$ and we assume that the initial marking is $m_i(0)=C_i, m_2(0)=0$ (see Fig. 4).

The link between the intersection cell and adjacent lane or intersection cells is modeled by continuous transitions $t_{IN}$, $t_{OUT}$, $t_{IN}$ and $t_{OUT}$ that describe respectively the input and output of the intersection cell with paths $a$ and $b$.

IV. MODEL OF A SIGNALIZED INTERSECTION AREA

The case study is a TN (see Fig. 5) located in the urban area of Bari, Italy and is composed by 6 links $L_i$ ($i=1,\ldots,6$) with length $l_1=40m, l_2=45m$ and $l_3=60m$, respectively. The links capacities are $C_{L1}=16$ PCUs, $C_{L2}=C_{L3}=9$ PCUs, $C_{L4}=C_{L5}=24$ PCUs and $C_{L6}=12$ PCUs, derived assuming that one PCU is $l_0=5m$ long and considering the lanes in each link.

![Figure 1. Example of urban area comprising two junctions pertaining to a common semaphoric cycle.](image1)

![Figure 2. The FOHPN model of two adjacent lane cells.](image2)

![Figure 3. The FOHPN modelling a random lane interruption.](image3)

![Figure 4. The FOHPN model of the intersection cell.](image4)
Figure 6 shows the FOHPN modeling the case study, using the elementary modules in Figs. 2 to 4, where we consider three cells for each lane in every link, with lane capacities depicted in the figure. Considering the number of lanes and the capacity of each link, $L_1$ and $L_6$ are modeled by 12 continuous places each, while $L_{-2}$, $L_3$, $L_4$ and $L_5$ include 6 continuous places each. It holds: $P_c={}^{1}_{p_{71}}, P_{c}={}^{1}_{p_{74}}, T_c={}^{1}_{t_{11}}, T_3={}^{1}_{t_{44}}, T_5={}^{1}_{t_{46}}, T_d={}^{1}_{t_{45}}, T_o={}^{1}_{t_{42}}, T_s={}^{1}_{t_{41}}$. In particular, transitions in the FOHPN are as follows: $p_i \in P_c$ with $i=1,...,24$ and $i=37,...,60$ model lane cells and their capacities, respectively; places $p_i \in P_c$ with $i=25,...,36$ and $i=61,...,66$ model the six intersection cells in the crossing area and their capacities, respectively; places $p_i \in P_c$ with $i=67,...,74$ are control places monitoring the vehicles that have entered the TN links (places $p_{67}, p_{68}, p_{70}, p_{73}, p_{74}$) and those that have left the intersection (places $p_{69}, p_{71}, p_{72}$); places $p_i \in P_d$ with $i=75,...,122$ model random lane interruptions. Transitions in the FOHPN of Fig. 6 have the following meanings: all $t_i \in T_c$ model input and output transitions of lane and intersection cells; all $t_i \in T_5$ model random lane interruptions and restorings. Hence, $t_{11}, t_{13}, t_{15}, t_{28}$ and $t_{32}$ are input transitions and $t_{30}, t_{19}, t_{27}$ are output transitions. Transitions $t_{16}, t_{18}, t_{19}$ and $t_{24}$ model vehicles entering a crossing section and are controlled by traffic lights.

The firing speeds of the input continuous transitions of Fig. 6 are in Table I. Three scenarios are considered: weekday daytime at rush hour (Scenario 1), weekday daytime (Scenario 2) and weekend (Scenario 3), in decreasing congestion order. Note that while $L_1$ and $L_6$ display similar traffic conditions, $L_3$ is under-saturated, being dedicated to public vehicles. In particular, the maximum firing speed of each input transition for such scenarios equals the average number of vehicles entering each lane, a known parameter based on traffic data. Table I also reports the firing speeds of the lane and intersection cells continuous transitions of the FOHPN of Fig. 6. Assuming that $v=40$ km h$^{-1}$ is the vehicle average speed in the TN (based on experimental evidence), the maximum firing speed of each such transition is determined as the ratio between $v$ and the length of the cell to which the considered transition is in input. Note that for the sake of simplicity we assume $\lambda_i=\lambda_j=0$ in the random lane interruption modules as in Fig. 3, so that lane interruptions are absent. Further, note that when lane changing is available (i.e., for $L_1$, $L_2$ and $L_3$), this is performed by 90% of vehicles, i.e., the fraction of vehicle flow not changing lane is $r=0.9$ (see Fig. 2). Moreover, the initial marking and capacities of the FOHPN are depicted in Fig. 6.

The FOHPN arc weights in the crossing sections of Fig. 6 are chosen so as the rate of vehicles traveling from $L_i$ to $L_j$ are $\beta_{ij}=-\beta_{ji}=-0.8$ and $\beta_{ij}=\beta_{ji}=0.2$ (see Fig. 6, where parameters $a=b=0.5$ represent the rates of vehicles travel to a pre-determined link or able choose between two directions).
The discrete places of the traffic light firing time of each transition describe the succession of red, yellow and green phases. The light model represents phases and the discrete transitions their duration during such phases. Figure 8 shows the timing plan phases, phases 5 and 8, omitted in Fig. 7 since no stream is allowed times, the signal timing plan comprises 8 phases, including 3 numbers from 1 to 5. Considering amber and inter-green

<table>
<thead>
<tr>
<th>Transition</th>
<th>Input transitions</th>
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</thead>
<tbody>
<tr>
<td>t1, t4, t5</td>
<td>Scenario 1: [0, 0.2860], Scenario 2: [0, 0.143], Scenario 3: [0, 0.0715]</td>
</tr>
<tr>
<td>t12, t24, t25</td>
<td>Scenario 1: [0, 0.2220], Scenario 2: [0, 0.111], Scenario 3: [0, 0.0555]</td>
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</table>

Lane and intersection cells transitions

<table>
<thead>
<tr>
<th>Transition</th>
<th>Input transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1, t4, t5, t6</td>
<td>[0, 1.1100]</td>
</tr>
<tr>
<td>t9, t10, t11, t12, t17</td>
<td>[0, 0.7400]</td>
</tr>
<tr>
<td>t18, t19, t20, t21, t24, t25, t26</td>
<td>[0, 0.5600]</td>
</tr>
<tr>
<td>t27, t28, t29, t30, t31, t32, t33, t34, t35</td>
<td>[0, 2.2200]</td>
</tr>
</tbody>
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Table I

<table>
<thead>
<tr>
<th>Phase</th>
<th>Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8</td>
<td>31 4 1 4 2 2 4 2</td>
</tr>
</tbody>
</table>

Figure 7. Streams in the signal timing plan of the intersection in Fig. 5.

Figure 8. The signal timing plan of the intersection in Fig. 5 (□ = green, □ = amber, □ = red).

Figure 9. The PN modelling the traffic lights of the intersection in Fig. 5.

The streams allowed during the signal timing plan phases of the considered TN are depicted in Fig. 7 and labeled with numbers from 1 to 5. Considering amber and inter-green times, the signal timing plan comprises 8 phases, including 3 amber phases (i.e., phases 2, 4 and 7) and 2 lost times (i.e., phases 5 and 8, omitted in Fig. 7 since no stream is allowed during such phases). Figure 8 shows the timing plan phases, their duration τ_i with i=1,...,8, and the cycle time CT.

To model the traffic lights we use the PN formalism presented in the literature [6]. The discrete places of the traffic light model represent phases and the discrete transitions describe the succession of red, yellow and green phases. The firing time of each transition t_j∈T_j is FT_j. The signal timing plan, described by Figs. 7 and 8, is realized by three traffic lights, each one with three phases modeled by three places representing the red, yellow and green phases, respectively.

Hence, to model the cycle phases, nine discrete places are necessary with three control places selecting the active streams (see Fig. 9): p_{127}, ..., p_{134}∈P_a. A token in p_{123} enables t_4 and t_5 to rule the vehicles of L_1. Moreover, p_{124}, p_{125} and p_{126} describe the state of green, yellow and red, respectively, for streams 1 and 2 in Fig. 7. The discrete transitions t_{92}, ..., t_{99}∈T_D model the timing plan: the firing times FT_{92}, FT_{93}, FT_{94} are the time intervals of green, yellow and all red (clearance time) of the traffic light of L_1. Hence, to describe the signal timing plan in Fig. 8 by the PN in Fig. 9, we select FT_{92}−τ_6 (the duration of the green signal controlling L_1), FT_{93}−τ_7 (the length of the amber phase ruling L_2) and FT_{94}−τ_8 (the subsequent lost time).

Similarly, we describe the traffic light ruling L_6 and modeled by p_{127}, p_{128} and p_{129} with FT_{95}−τ_1 (the green signal duration for L_6) and FT_{96}−τ_2 (the duration of the amber phase for L_6). In addition, the traffic light controlling L_3 is modeled by p_{123}, p_{124} and p_{125} with FT_{97}−τ_3 (the duration of the amber phase for L_3), FT_{98}−τ_4 (the amber phase duration for L_3). Further, t_{99} models the inter-green signal duration subsequent to the allowed stream originating by L_3, hence FT_{99}−τ_5.

The start of the cycle is with a token in p_{123} and p_{124} (green for L_1). After the yellow for L_1 (t_5 fired so that a token is in p_{122}) the red phase begins for L_1, t_6 fires and a lost time begins. Hence, after FT_{94} t_6 fires, the green phases start for L_6 and L_3 (tokens in p_{127}, p_{130} and p_{131} and p_{132}). The green for L_6 is shorter than for L_3: a token in p_{129} with p_{134} unmarked means that L_6 is in the red phase and L_3 is in the green or yellow phase. When a token moves in p_{134}, t_9 is enabled and the second all red phase starts. When t_9 fires (after the lost time FT_{99}), the green phase starts for L_1 and the cycle begins again.

V. SIMULATION RESULTS

The FOHPN in Fig. 6 of the case study in Fig. 5 was implemented and simulated in MATLAB. The model modularity suggests using this efficient software, allowing to model large systems. Moreover, such a matrix-based software is particularly suitable for simulating FOHPN dynamics based on the matrix formulation of the marking update. As regards the model implementation, at the beginning of each macro-period the program defines and solves (1), based on knowledge of the system state x(τ_i). Note that solution v*(1) is chosen as the one maximizing the performance index J=1^T v, that is the IFS vector is chosen in (1) to maximize the system throughput. This means that the firing speed of continuous transitions is either 0 or V_{max}. Having obtained the speed values v(τ_i), the program determines matrices A(τ_i) and B(τ_i), as well as the occurrence time and type of the next macro-period. Hence, (5) is solved and the procedure iterated.

To validate the FOHPN model, we compare the TN dynamics in the three scenarios with an alternative CTPN formalism proposed in [6] and applied to the same case study. In the CTPN approach the traffic light controller is the same as the one described previously, but the TN is described by a modular CTPN where tokens represent vehicles, the tokens colors model vehicle paths, places are cells accommodating one vehicle and discrete transitions model the vehicle flow.
The simulations run time is $T=1540$ s, so that 22 timing plan cycles are considered, but the first 2 are regarded as transient. Simulations start from the initial state in which no vehicle is present in the TN. The FOHPN model generates as outputs for each input link $L_i$ with $i=1,3,6$, the number of vehicles $n_i(k)$ with $i=1,3,6$ at the beginning of the $k$-th cycle green phase.

Variable $n_i(k)$ for $L_i$ with $k=1,\ldots,20$ is reported in Figs. 10, 11 and 12 for Scenarios 1 to 3, respectively, for the proposed FOHPN model (results in light grey) and for the CTPN model (results in black). Note that $L_i$ has a similar traffic congestion as $L_6$, while $L_2$ is very under-saturated, so its dynamics is omitted. The figures show that the FOHPN results are consistent with the CTPN microscopic model dynamics, confirming the FOHPN effectiveness. Remarkably, the fluid approximation by FOHPN leads to similar results as the more precise CTPN model, even with extremely under-saturated conditions, as in Scenario 3 (see Fig. 12). The maximum percentage relative error for $n_i(k)$ in $L_i$ with $i=1,3,6$ and $k=1,\ldots,20$ obtained by FOHPN with respect to the corresponding values obtained by CTPN is 5%, corresponding to a 1.3 PCUs maximum absolute error (about one vehicle is incorrectly simulated by the FOHPN approach).

Summing up, the FOHPN model presents three main advantages with respect to the related literature. First, the required parameters are the number of vehicles entering each road lane and their average rate (to determine the firing speeds of the input continuous transitions). These can be directly measured at each input branch of every intersection. Second, the model is modular and easy to build on the basis of the TN topology. Third, the code to simulate real intersections can be directly generated in order to test both fixed signal timing plans or actuated control strategies. Comparing the approach with others based on ordinary PN (e.g., [4, 8]) or high order HPN (e.g., [5]), FOHPN provide an intermediate level of representation, increasing the PN descriptive power while retaining the ease of obtaining input parameters. With respect to [5], we introduce lane interruptions and intersection cells, resulting in a more accurate TN model. With respect to the CTPN model in [6], the FOHPN formalism provides a more aggregate formulation, reducing the dimension of the state space, while sharing the CTPN accurateness in representing the TN dynamics. Finally, FOHPN are suitable for real time optimization of signal timing plans, a future research subject.

VI. CONCLUSIONS

We present a First Order Hybrid Petri Nets (FOHPN) model for urban traffic networks. The model is macroscopic, accurate and provides an inexpensive to develop simulation software. Exploiting some characteristics of FOHPN we define a modular model, where vehicles are represented by a fluid that flows in the system and is subject to random and unpredictable discrete events. To show the technique effectiveness, a case study describing an intersection located in Bari, Italy, is simulated in MATLAB. Validation is performed by comparison with a microscopic model based on Colored Timed Petri Nets (CTPN) proposed by some of the authors. FOHPN, although providing a more aggregate (but simpler) system model than CTPN, lead to comparable results in terms of accuracy. For simplicity a single junction is considered as case study, but the model modularity allows representing a large number of intersections, simply connecting FOHPN sub-models. Future research will consider real time optimization of signal timing plans, e.g., determining real time optimization of

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