Genetic algorithms for supply-chain scheduling: A case study in the distribution of ready-mixed concrete

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Abstract

The coordination of just-in-time production and transportation in a network of partially independent facilities to guarantee timely delivery to distributed customers is one of the most challenging aspect of supply chain management. From a theoretical perspective, the timely production/distribution can be viewed as a hybrid combination of planning, scheduling and routing problems, each notoriously affected by nearly prohibitive combinatorial complexity. From a practical viewpoint, the problem calls for a trade-off between risks and profits. This paper focuses on the ready-mixed concrete delivery: in addition to the mentioned complexity, strict time-constraints forbid both earliness and lateness of the supply. After developing a detailed model of the considered problem, we propose a novel meta-heuristic approach based on a hybrid genetic algorithm combined with constructive heuristics. A detailed case study derived from industrial data is used to illustrate the potential of the proposed approach.

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1. Introduction

Recently, production industry is experiencing a strategic evolution toward the decentralization of many production activities. Due to the various, extremely challenging problems related to collaboration, cooperation, information sharing, and synchronization of logistic and production activities, research areas such as
supply chains, virtual enterprises, global manufacturing networks and similar forms of organization have gained a prominent role in the field of industrial automation (Viswanadham, 2002). Supply chains can be viewed as dynamic networks of partially independent production centers that agree to collaborate for pursuing both individual and collective aims. For instance, independent companies that are able to provide complementary services for the production of a given good may take a significant advantage by synchronizing their activities to reduce product lead times or costs. From the logistic viewpoint, the management of supply chains involves a set of complex and interdependent combinatorial problems (e.g. acquisition of raw materials, scheduling of production facilities, routing of transport vehicles, etc.). Each of the mentioned logistic problems suffers from a nearly prohibitive combinatorial complexity, even when considered as independent from the other ones. There is also a strong need for approaches that are capable of finding satisfactory solutions in short computation times, since these environments are continuously subject to unpredictable dynamic changes (new orders, delays, failures) that may require a sort of continual revision of the planned solution.

In this paper, we consider the supply chain for the production and distribution of ready-mixed concrete. The supply chain consists of a network of independent and distributed production centers serving a set of customers distributed across a predefined geographical area surrounding the nodes of the supply chain. Most production centers in the chain own a fleet of trucks to deliver the produced good to the customers, but a few centers do not own a local fleet for transportation and explicitly rely on the other nodes for the delivery of their production. By joining the supply chain, each production center agrees to convey the received demands to a central planning system, which is in charge of scheduling the production on the various centers in order to optimize the operation of the entire supply chain. This means that after the optimization, the company producing a given demand may no longer be the one that received the order, if this leads to a better overall schedule. The considered problem is made particularly challenging by the fact that the produced good is a perishable material that has to be mixed on-demand and delivered within strict time-windows to customers’ locations. The individual goal of each production center is to accept and satisfy the maximum number of requests, guaranteeing the timeliness of the deliveries at the minimum overall cost. Some of the received requests may partially exceed the capacity of the contacted production center, but the center may still accept the request and redirect part of the order to another node of the supply chain, or outsource it to an external company. Note that the goal of each production center is composed of inherently conflicting aspects, because on the one hand the maximum utilization of resources implies the reduction of idle and waiting times, and on the other hand these time intervals are the only actual safety margins that make the schedule tolerant to transportation delays or other unexpected circumstances. In any case, since the costs associated to late deliveries can be extremely high, an effective tradeoff between schedule robustness to perturbation and production costs must be necessarily found.

At present, many companies tend to rely on skilled operators that set up an initial planning of production, also estimating which customer’s request can be accepted, and which has to be rejected (Matsatsinis, 2004; Feng et al., 2004). Other companies prefer to plan their operations on short production horizons, sacrificing the optimization on longer horizon to achieve a reduced risk of delayed delivery (Tommelein and Li, 1999). However, to the authors’ knowledge, there is no specific approach in the literature that addresses the ready-mixed concrete delivery problem in its full complexity, dealing with the complex constraints and a large number of specific properties that must be considered in practice. The main contribution of this paper is the development of an effective strategy to systematically model and solve the just-in-time production and supply of ready-mixed concrete in an efficient, reliable, and systematic way, so as to bridge the gap between industrial practice and technical research. The development of such an effective approach also improves the sustainability of the supply chain management solutions by increasing the utilization of equipment and by decreasing the demand on scarce resources.

The first step of our approach is based on the development of a detailed mathematical model of the considered problem. The development of an extensive model, explaining the available decision variables
and the main constraints of the problem, constitutes the first important contribution of this paper with respect to the related literature that is mainly focused on simplified formulations taking into account only a part of the considered problem. The second step of our approach consists of the development of an algorithm based on genetic algorithms (GAs), a class of modern problem-solving meta-heuristics that seem particularly suited to the considered problem. GAs are heuristic search techniques inspired from the principles of survival-of-the-fittest in natural evolution and genetics. They are known to search efficiently in a large search space, without explicitly requiring additional information (such as convexity, or availability of derivative information) about the objective function to be optimized. For this reason, in the last decade, they have been applied to many (combinatorial) problems, including scheduling and vehicle routing applications that are partially related to our problem. However, GAs are generally slow, and the average time that a well-configured GA would need to search for a satisfactory solution of the entire supply-chain problem may be too high for practical use in a real industrial context, where the decision algorithm must provide a solution in relatively short times. This is also the case for the problem considered in this paper. For this reason, we use the GA only to address a part of the whole problem in our research. Namely, we use the GA to perform demand-to-production center assignment, and the production sequencing at each center, while the remaining part of the whole scheduling problem is handled by constructive heuristic algorithms. This approach leads to a hybrid evolutionary algorithm in which the GA constitutes the core of the search strategy, while multiple heuristic rules called in specific circumstances contribute to reconstruct a feasible solution that satisfies all the constraints and objectives. This hybrid approach enables us to address the extremely complex scheduling problem of an entire supply-chain for just-in-time production by explicitly taking into account all the constraints and requirements of a real-world scenario. Further, the proposed model and the solution strategy are fairly general, and can easily be extended to address other just-in-time distributed production and delivery problems of perishable goods, similar to ready-mixed concrete.

The paper is organized as follows. Section 2 gives an overview of the related literature, while Section 3 describes the mathematical model of the considered problem. Section 4 introduces the proposed hybrid meta-heuristic approach, illustrating the main GA-based search engine, and the additional schedule construction heuristics in separate subsections. Section 5 describes a case study, summarizing the main results of our approach in comparison with other scheduling heuristics. It also provides a detailed analysis of the tolerance of the optimized schedules to delivery delays. Finally, Section 6 provides the concluding remarks.

2. Literature overview

Producing and distributing ready-mixed concrete is a complex logistic problem that involves several interdependent assignment and scheduling problems. Moreover, the specific characteristics of the produced material and its utilization in construction entail a large number of additional technical constraints that must be taken into account. An overview of the main characteristic of cement production and delivery is provided in (Tommelein and Li, 1999). It surveys various types of vertical supply chain organizations that can be adopted by production companies and customers to pursue their objectives. Tommelein and Li remark that the most common case is the one where the batch plant also delivers the mix to the contractor’s project site, i.e. the case considered in this paper. The problem of cement delivery is also considered in (Matsatsinis, 2004), where a decision support system is proposed to address the routing of two types of vehicles: the cement carriers for the delivery of the concrete to customers’ sites, and the pumps that may be necessary in some sites to unload the cement from the trucks. Their proposed solution is based on an approach that iteratively improves an initial assignment done by plant managers.
It can be easily noted that the just-in-time supply problem considered here also shares many common aspects with a large number of related logistic problems that have been considered in the literature. For instance, the assignment of trucks for transportation can be modeled as a vehicle routing problem (VRP), which is amongst the most thoroughly investigated problems of operation research. The version of the VRP most related to our case study concerns the routing of a fleet of vehicles with time-window constraints and is known as Multi-Depot Multi-Vehicle Routing Problem with Time-Windows. Due to the combinatorial complexity of this problem, the available literature focuses on heuristic approaches capable of achieving satisfactory solutions in acceptable times. Comprehensive surveys about exact and heuristic methods to deal with the VRP are available in (Laporte et al., 2000; Toth and Vigo, 2002; Marinakis and Migdalas, 2003). A study on vehicle capacity planning system that is considerably related to our problem is provided in (Lee et al., 2003), where the authors model container transportation as a vehicle routing problem with time-window constraints. The authors propose a heuristic approach based on Tabu Search, which is able to determine solutions that are significantly better than those provided by the existing rules adopted by the transport company considered.

In recent years, there has been an increased interest in the application of evolutionary approaches (Michalewicz, 1996) for tackling combinatorial complexity in solving various supply chain problems. Chan et al. (2004) investigate the application of multi-criterion genetic optimization for order distribution in a collaborative supply chain. Vergara et al. (2002) use evolutionary algorithms to optimize material flow in supply chains. Jeong et al. (2002) apply genetic algorithms for forecasting important supply chain management quantities, such as demand volume. Zhou et al. (2003) apply GAs to allocate customers to warehouses. Evolutionary approaches have also been used in problems related to supply chain scheduling considered in this paper. Lee and Choi (1995) apply GAs for allocating set of independent jobs with delivery time constraints to a set of distributed plants. The proposed method provides nearly optimal solutions as confirmed by a comparison with an exact algorithm. Serifoglu and Ulusoy (1999) consider the problem of scheduling independent jobs on identical parallel machines to minimize earliness/tardiness with a GA. They find that the GAs outperform a neighborhood exchange search algorithm in larger-sized, more difficult problems, providing improvements that increase with the problem size. The scheduling of identical parallel machines with a GA has been considered also in (Min and Cheng, 1999), where the objective is formulated as the minimization of the makespan, and the performance of the GA is compared against Simulated Annealing and other available heuristics. Also here, the proposed GA outperforms the other methods suggested in the paper for comparison.

Recently, some authors have proposed the application of GAs to solve some of the problems related to concrete production and delivery. Garcia et al. (2002) consider the problem of scheduling a single production plant in order to satisfy delivery time constraints. They propose two approaches, an exact method suitable only for very simple cases, and a GA for instances of more realistic size. The paper does not address a realistic application scenario, as it considers only a single depot, ignores limited resources for transportation, and considers instances that are significantly smaller than those used in our case studies. In another recent research work, Feng et al. (2004) focus on scheduling for a single depot, equipped with a fleet of vehicles with identical capacity and a fixed (customer and depot independent) loading/unloading times. The authors propose the use of a GA for searching a production sequence that maximizes a predefined performance index (taking into account truck waiting times and a penalty for violating the unloading continuity of multi-truck orders) that is evaluated by discrete-event simulation of the operations of the fleet of vehicles. However, the assumption of identical production and loading/unloading times for each order is not acceptable in our real-world case studies. Moreover, the largest instance considered by Feng et al. (composed of a single depot processing 9 demands fractioned in 22 distinct sub-deliveries performed by a fleet of 20 trucks) is significantly smaller than the cases considered in this paper (five depots receiving 40–70 demands, determining 200–450 sub-deliveries assigned to a fleet of about 50 trucks).
3. Modeling the ready-mixed concrete supply chain

We now give a more detailed description of the concrete delivery problem examined in this paper. First of all, it should be mentioned that once the production center adds water to the mix of dry materials, the concrete has only about two hours (unless specific chemical retarders are employed) before the hydration process forms a gel that, if disrupted, would compromise the ultimate strength of the concrete. Thus, large orders require a strictly uninterrupted supply of concrete in order to avoid dangerous construction joints (Tommelein and Li, 1999). A delivery sequence for uninterrupted supply at a site requires a loaded truck to be available at the site when the preceding truck has ended the unloading. In some circumstances, there can be extremely large orders that involve a considerable number of trucks and thus tie up most of production plant’s and vehicle fleet’s capacity. Since concrete should be placed no later than two hours after the addition of water, travel from the batch plant to a site should not take much more than an hour or so. Therefore, a plant’s operating radius tends to be limited based on the nature and condition of haul roads. The time a ready-mix truck may sit in traffic during rush hours is a significant consideration when scheduling site deliveries. On-time delivery of concrete is essential to a customer. If a truck arrives early, the concrete placement crew may not yet be ready. If a truck arrives late the continuity of the unload is violated, and if the delay exceeds the concrete setting time the entire load has to be disposed.

We consider a network of \( D \) suppliers or Production Centers (PCs) located in a given geographical area. Each PC is equipped with a single loading dock, where the produced cement is loaded on the trucks for its delivery. Each loading dock can service only one truck at a time. Each order or fraction of an order is produced at the time the truck assigned to it is available at the loading dock for loading the material. The total loading time for a truck consists of a fixed part (independent of the loading rate) and a flexible part that depends on the required volume and the loading rate of the loading dock. Moreover, there is no significant setup time depending on the type of concrete mixed in two successive load operations.

An order consists minimally of a delivery moment (date and time), type of concrete, required amount and delivery location. Additionally, a customer may require explicitly that an order is produced by a specific PC or that it must not be supplied by some of the PCs of the chain. Several other attributes of customer’s resources at the unloading site must also be taken into account. Namely, each customer can unload at a given maximum rate, and may specify a maximum amount of product per single delivery. Moreover, the customer may require a truck to arrive in advance with respect to the requested delivery time for some logistic reasons at the construction site. In other words, the customer may require that the truck waits a predefined time interval at the unloading site prior to unloading. Some orders do not need a delivery, as the customer picks up the concrete himself. Clearly, these orders must be taken into account only in the PCs scheduling.

Some characteristics of the trucks are also fundamental parameters of the whole scheduling problem. In fact, the delivering trucks have a limited capacity, and a maximum unloading rate. Some specific types of concrete require that a certain fraction of the truck’s capacity is left empty, thus reducing the actual capacity of the truck for that type of cement. Trucks are parked in specific base locations at the PCs from which they start every morning, and to which they must return every evening. A vehicle can be used for a predefined day shift, so any delay or additional job must be paid additionally. Contrary to many other routing problems, each truck can service only one order at a time. It is not possible to service multiple small orders by the same truck during the same delivery. Hence, a small order often implies that a truck will be only partially loaded during the delivery. As for the loading process, only one truck at a time can be unloaded at the delivery location (if two trucks have arrived at the location, one must wait until the other one is unloaded). Finally, whenever needed, it is possible to hire a number of additional vehicles from external companies.

We model the integrated supply chain scheduling as a cost minimization problem. For convenience, the list of all the symbols and acronyms used in our model is reported in Appendix A. We suppose that at a
given decision time, \( R \) (request) demands from different customers have been received, and have to be assigned to \( D \) (depot) different PCs. If a demand exceeds the capacity of a single truck, it is divided in a number of sub-demands (jobs), which will be delivered to customers. Thus, we introduce the following indices:

- \( d \in \{1, \ldots, D\} \) depot-related index
- \( r \in \{1, \ldots, R\} \) customer or demand-related index
- \( i \in \{1, \ldots, N\} \) job-related index relative to the job. \( N \) is total number of jobs to perform. The sub-demands are arranged in sequential orders, so that the index \( i \) can be interpreted as follows:

\[
\begin{align*}
  i \in \left\{ 1, \ldots, Z_1, Z_1 + 1, \ldots, Z_2, Z_1 + Z_2 + 1, \ldots, Z_3, \ldots, \sum_{r=1}^{R} Z_r + 1, \ldots, N \right\},
\end{align*}
\]

where \( Z_r \) is the number of sub-deliveries in which a request exceeding a truck’s capacity is divided. Furthermore, we indicate with \( f_r \) and \( l_r \), the first and last job in the demand \( r \), respectively

- \( k \in \{1, \ldots, K\} \) truck-related index. \( K \) is the total number of trucks \( (K = K_c + K_o) \), where \( K_c \) is the number of trucks of the company, and \( K_o \) is the number of additionally hired trucks

- \( m \in \{1, \ldots, M_k\} \) task-related index. A task of a truck is the delivery of a job to its destination. \( M_k \) is the maximum number of tasks allowed to a single truck \( k \)

If necessary, additional trucks may be hired and part of the production requests can be outsourced to external companies at an additional cost. We assume that if the production of a job is outsourced, the job will be directly delivered to the customer at the specified time, i.e. our model does not deal with the delivery of outsourced production. On the contrary, we do consider the scheduling of hired trucks for the delivery of jobs that cannot be handled by the internal fleet as a part of our problem. Our model then considers the following decision variables:

- \( X_{ikm} \in \{0, 1\} \) if the job \( i \) is assigned to truck \( k \) as \( m \)th task, \( X_{ikm} = 1 \), otherwise \( X_{ikm} = 0 \)
- \( Y_{id} \in \{0, 1\} \) if job \( i \) is produced at the depot \( d \), \( Y_{id} = 1 \) and \( i \in I_d \subseteq \{1, \ldots, R\} \), otherwise \( Y_{id} = 0 \)
- \( Y_{oi} \in \{0, 1\} \) if the production of job \( i \) is outsourced, \( Y_{oi} = 1 \), otherwise \( Y_{oi} = 0 \)

The cost function is composed of three terms:

\[
C = C' + C'' + C'''.
\]

\( C' \) takes into account the transportation costs, in terms of total distance traveled by the fleet of trucks to deliver all the produced jobs (see (2)). The first term in (2) takes into account the sum of all the distances from PCs to customer sites. The second term accounts for the return trip to the PC for the next job, while the third term accounts for the cost of reaching the PC supplying the first job of the truck, if it differs from the base location, and the cost of returning to the base-location at the end of the working day. From a global viewpoint, the production of the entire supply chain should be organized so as to minimize these delivery costs

\[
C' = \frac{CP}{V} \left( \sum_{i=1}^{N} \sum_{d=1}^{D} Y_{id} A(r_i, d) + \sum_{k=1}^{K} \sum_{m=1}^{M-1} \sum_{j=1}^{N} X_{ikm} X_{jk(m+1)} \left( \sum_{d=1}^{D} Y_{jd} A(r_i, d) \right) \right)
\]

\[
+ \sum_{k=1}^{K} \left( A(DP_k, DF_k) + A(DL_k, DP_k) \right) \right).
\]

\( C'' \) takes into account the loading and unloading waiting times (see (3)). As mentioned before, this is a critical variable of the problem. The waiting times measure how much in advance the truck must be ready for the operations at a PC or at a delivery site with respect to the actual start of the loading or unloading oper-
ation. In principle, waiting times should be minimized because they typically represent a loss (the more the waiting times, the lower the resource utilization). However, waiting times are the only safety margins of a given solution, since a schedule with reasonable waiting times allows a truck to perform its operation also in presence of delays (e.g. transport delays due to traffic)

\[ C'' = CA \left( \sum_{k=1}^{K} \sum_{m=1}^{M} LWT_{km} + \sum_{i=1}^{N} UWT_i \right). \]  

\( C'' \) accounts for the additional costs related to outsourced production, hired trucks and the overtime work for some truck drivers (see (4)). Solutions requiring a different number of outsourced jobs or hired trucks can be found for any given set of demands. The more the schedule is optimized, the lower the amount of requested outsourcing. Of course, this contrasts with the just mentioned safety margins, because in general optimized schedules have very tight safety windows

\[ C'' = PT \left( \sum_{i=1}^{N} Y_{oi} \frac{Q_i}{Z_{r(i)}} \right) + HC \cdot K_o + XTR \left( \sum_{k=1}^{K} \max \{0, STWD_k - T^{start}_k \} + \max \{0, T^{6}_{kM_k} - ENDWD_k \} \right). \]

The optimization model is subject to a considerable number of assignment and timing constraints that will be introduced and commented separately in the next subsections. For the sake of clarity, problem constraints are numbered separately.

### 3.1. Assignment constraints

Every job can only be assigned to a production depot or outsourced once, and so

\[ (c1) \quad \forall i \in \{1, \ldots, N\} \quad \sum_{d=1}^{D} Y_{id} + Y_{oi} = 1. \]  

Similarly, each job can only be assigned once to either a truck of our fleet or to an hired one

\[ (c2) \quad \forall i \in \{1, \ldots, N\}, \quad \begin{cases} \text{if } Y_{oi} = 0, & \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} = 1, \\ \text{if } Y_{oi} = 1, & \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} = 0. \end{cases} \]  

Finally, jobs must be assigned to trucks sequentially (in other words guaranteeing that for the \( n \)th task of any truck \( k \) all the preceding tasks are assigned and all the succeeding task are not assigned)

\[ (c3) \quad \forall k \in \{1, \ldots, K\}, \quad \forall m \in \{1, \ldots, M_k - 1\} \quad \sum_{i=1}^{N} X_{ik(m+1)} \leq \sum_{i=1}^{N} X_{ikm} \leq 1. \]

### 3.2. Computation of operation and travel times

The considered supply problem has a number of constraints related to some start or end times of specific truck operations. In order to clearly introduce such constraints, Fig. 1 defines the typical sequence of operations for a truck, and specifies the associated time intervals. In particular, once a truck is available at the PC, it has to (i) wait for the start of loading operations, (ii) load the material, (iii) travel to destination, (iv) wait for unloading (including the customer-specified fixed time), (v) unload, and (vi) return to a PC or base location.
The job loading time $LT_i$ depends only on the depot loading rate $LR_d$ and on job size $Q_r/Z_r$:

$$\forall i : f_r \leq i \leq l_r \quad LT_i = \sum_{d=1}^{D} Y_{id} \cdot \left( FLT_d + \frac{Q_r}{Z_r} \cdot \frac{1}{LR_d} \right),$$

while the source to destination traveling time $SDT_i$ for job $i$ is computed as

$$\forall i : f_r \leq i \leq l_r \quad SDT_i = \frac{\sum_{d=1}^{D} Y_{id} \cdot A(r_i, d)}{V}.$$

Finally the traveling time between the destination of job $i$ and the source of job $j$ $DST_{ij}$ is computed as follows:

$$\forall i : f_r \leq i \leq l_r \quad DST_{ij} = \sum_{d=1}^{D} Y_{jd} \cdot \frac{A(r_i, d)}{V}.$$

### 3.3. Delivery time-window related constraints

Since each customer assigns the time-window for delivery, a preliminary verification of data consistency should be carried on. In particular, it should be inspected if

$$\forall r \in \{1, \ldots, R\} \quad LDT_r - EDT_r \geq Z_r \cdot UT_i,$$

i.e. that the customer assigned delivery time-window is sufficiently wide to allow the completion of unloading operations at the available unloading rate of the customer $r$. If this requirement is violated, a warning is issued to the customer. Moreover, accepted jobs must be scheduled meeting the following constraints:

(c4) $\forall i : f_r \leq i \leq l_r \quad ELT_i - SLT_i \geq LT_i,$

This condition guarantees that the time-window allocated to each single job is large enough to allow the completion of the loading at the depot $d$

(c5) $\forall i : f_r \leq i \leq l_r \quad SLT_i \geq LDT_r - (l_r - i) \cdot UT_i - Tset_r.$
This constraint guarantees that the start of loading time-window is not so early that the concrete may solidify before it is completely unloaded
\[(c6) \forall i : f_r < i \leq l_r \quad \text{ETL}_i \leq \text{LDT}_r - \text{SDT}_i - \text{Fix}_r - (l_r - i + 1) \cdot \text{UT}_i.\]  
This constraint affects the end of loading time-window, ensuring that the end of the loading cannot be so late that the sequence of remaining jobs will not be completed within customer-specified time-window
\[(c7) \forall d \in \{1, \ldots, D\}, \forall i_1, i_2 \in \Gamma_d, i_1 \neq i_2 \quad \text{ELT}_{i_1} \leq \text{SLT}_{i_2} \lor \text{SLT}_{i_1} \geq \text{ELT}_{i_2}.\]
This constraint forbids the overlap of loading time-windows at the same depot.

### 3.4. Single truck related constraints

Truck operation times must be computed according to the following relations, and meeting the constraints introduced subsequently
\[\forall k \in \{1, \ldots, K\}, \quad T^0_{k1} = \frac{A(DP_k, DF_k)}{V} + t^\text{start}_k.\]  
The start of task \(m\) for truck \(k\) can be related to the time \(T^0_{k1}\) when it begins its first task as follows:
\[\forall k \in \{1, \ldots, K\}, \forall m \in \{2, \ldots, M_k\}\]
\[T^0_{km} = T^0_{k1} + \sum_{i=1}^{m-1} \left( \text{LWT}_{kl} + \sum_{i=1}^{N} X_{ikl} \cdot (\text{LT}_i + \text{SDT}_i + \text{UWT}_i + \text{FIX}_i + \text{UT}_i) + \sum_{j=1}^{N} \sum_{j \neq i} X_{ikl} \cdot X_{jkl(m-1)} \cdot \text{DST}_{ij} \right),\]  
\[\forall k \in \{1, \ldots, K\} \quad \text{LWT}_{k1} = T^1_{k1} - T^0_{k0},\]  
\[\forall k \in \{1, \ldots, K\}, \forall m \in \{2, \ldots, M_k\} \quad \text{LWT}_{km} = T^1_{mk} - \sum_{i=1}^{N} X_{ikm(m-1)} \cdot T^0_{k(m-1)},\]  
\[\forall k \in \{1, \ldots, K\}, \forall m \in \{1, \ldots, M_k\} : \quad \text{LWT}_{km} \geq \text{MWT},\]  
\[T^1_{km} = T^0_{km} + \text{LWT}_{km},\]  
\[T^2_{km} = T^1_{km} + \sum_{i=1}^{N} X_{ikm} \cdot \text{LT}_i,\]  
\[T^3_{km} = T^2_{km} + \sum_{i=1}^{N} X_{ikm} \cdot \text{SDT}_i,\]  
\[\forall i : f_r \quad \text{UWT}_i = \max\left( \text{EDT}_r - \text{Fix}_r - \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T^3_{km}, \text{MWT} \right).\]

This condition states that for the first job related to a demand \(r\), the truck will have to wait longer than \(\text{MWT}\) only if it arrives earlier than the expected \(T^3_{km}\)
\[\forall i : f_r < i \leq l_r \quad \text{UWT}_i = \max\left( \sum_{m=1}^{M} \sum_{k=1}^{K} X_{(i-1)km} \cdot T^3_{km} - \text{Fix}_r - \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T^3_{km}, \text{MWT} \right).\]

Analogously, this condition specifies that for all the other jobs related to a demand \(r\), the truck will have to wait longer than \(\text{MWT}\) only if it arrives earlier than the end of the unloading of the previous job.
Similarly, this constraint means that the choice of $T_{km}^4$ must guarantee that $UWT_i$ is not less than $MWT$

$$ \forall k \in \{1, \ldots, K\}, \forall m \in \{1, \ldots, M_k\} \quad T_{km}^4 = T_{km}^3 + \sum_{i=1}^{N} X_{ikm} \cdot (UWT_i + Fix_r), \quad (26) $$

$$ T_{km}^5 = T_{km}^4 + \sum_{i=1}^{N} X_{ikm} \cdot UT_i, \quad (27) $$

$$ T_{km}^6 = T_{km}^5 + \sum_{i=1}^{N} \sum_{j=1 \atop j \neq i}^{N} X_{ikm} \cdot X_{jkm} \cdot DST_{ij}, \quad (28) $$

$$ \forall k \in \{1, \ldots, K\}, \forall m \in \{1, \ldots, M_k - 1\} \quad T_{km}^6 = T_{km}^0, \quad (29) $$

$$ \forall k \in \{1, \ldots, K\} \quad T_{km}^6 = \frac{A(DL_k + DP_k)}{V} + T_{km}^0. \quad (30) $$

The latter equation simply models the return of each truck to its base location. Truck scheduling constraints are defined as follows:

$$ (c8) \quad \forall i : f_r \leq i \leq l_r \quad SLT_i \leq \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^1 \leq ELT_i - LT_i = LLT_i. \quad (31) $$

This means that the loading of job $i$ must start and end within its loading time-window

$$ (c9) \quad \forall i : f_r < i \leq l_r \quad \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^1 \geq \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^6 + MWT. \quad (32) $$

This constraint guarantees that $T_{km}^1$ is chosen so as to ensure that the $LWT_{km}$ is greater than the minimum allowed safety margin $MWT$

$$ (c10) \quad \forall i : f_r \leq i \leq l_r \quad \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^1 \leq \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^4 - SDT_i - Fix_r - MWT. \quad (33) $$

Similarly, this constraint means that the choice of $T_{km}^1$ must guarantee that $UWT_i$ is not less than $MWT$

$$ (c11) \quad \forall i : f_r \leq i \leq l_r, \quad \sum_{d=1}^{D} \sum_{j=1}^{D} Y_{jd} = 0, \quad \text{and} \quad \sum_{d=1}^{D} Y_{jd} = 1, $$

$$ EDT_r + (i - f_r) \cdot UT_i \leq \sum_{k=1}^{K} \sum_{m=1}^{M} T_{km}^4 \leq LDT_r - (l_r - i + 1) \cdot UT_i. \quad (34) $$

This equation considers the case in which some of the jobs composing a demand are outsourced. In particular, the equation refers to the case in which the first jobs in the sequence (all those preceding job $i$) are outsourced. In such a case, job $i$ must be scheduled so that the preceding jobs are fully unloaded. All the other cases are considered in other equations (see c12).

### 3.5. Constraints between different trucks

The unloading of all the jobs (except the first one) composing demand $r$ must start exactly when the preceding one ends. This strict requirement guarantees the continuity of the unloading process at the construction site. This requirement is taken into account with the following constraint:

$$ (c12) \quad \forall i : f_r \leq i < l_r \quad \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^5 = \sum_{k=1}^{K} \sum_{m=1}^{M} X_{ikm} \cdot T_{km}^4. \quad (35) $$
4. Hybrid meta-heuristics for concrete production and delivery

Given a certain set of demands, finding a feasible schedule for the overall supply chain may be a task of variable but generally prohibitive complexity, which depends not only on the number but also on the specific characteristics of the demands. For instance, it can happen that the requests for a given day are well within the overall capacity of the network, but concentrated in areas that are out of the operating radius of most PCs, which consequently remain inactive. Similarly, requests may be conflicting in terms of closeness of their delivery times, so that it is not possible to schedule the loading operation on the various PCs guaranteeing the delivery of all the jobs in the requested time-windows. Many other problems may arise due to the limited fleet of vehicles for delivery. Moreover, there are a number of practical requirements that have not been explicitly taken into account in the proposed model for sake of simplicity, but may have a significant influence on the quality of the final solution. For instance, it is preferable to make all the efforts to keep the sub-demands for a single request assigned to the same PC, in order to avoid schedules with an excessive coupling between the PCs. Furthermore, some customers may explicitly require the production at a specific PC, thus constraining some of the decision variables $Y_{id}$ to predefined values. In order to take into account all the requirements of the problem, we propose a heuristic algorithm that decomposes the supply-chain problem in two separated sub-problems, which are tackled one after the other. In particular, in a preliminary stage, the demands are subdivided into separate jobs according to truck capacity and customer requirements. Then, the first problem concerns the assignment of jobs to PCs, and the associated scheduling of the simultaneous mixing and loading operations at each PC. When a loading operation is scheduled at a PC, it is assumed that a truck will be made available at the loading dock at the assigned loading start time. The second problem concerns the routing of the fleet of carriers, guaranteeing that a truck is located at a loading dock at a time a loading operation is scheduled to start. If the routing algorithm is not able to make a truck be available in time for the loading of the assigned job, an external truck is hired for this purpose at an additional cost. Clearly, this decomposition of the problem in two consecutive problems may lead to sub-optimal solutions, while ideally the problems of PC scheduling and truck routing should be jointly considered in a single global formulation. However, the decomposition allows us to achieve two sub-problems of reasonable complexity, and solve them effectively with relatively simpler algorithms. Moreover, it should also be remarked that the experimental investigation in our case study shows that the proposed search strategy is in general able to find solutions with generally short and evenly distributed waiting times, confirming that the internal fleet of trucks is in general appropriately exploited.

The two separated sub-problems are solved with different heuristic algorithms. The first sub-problem can be formulated as a search in a combinatorial space. Due to the size of the search space in realistic application scenarios, it is not feasible to apply enumerative techniques. Because GAs are known to search large spaces efficiently for satisfactory solutions, we apply a GA at this stage to optimize the assignment of demands to depots (the decision variables $Y_{id}$) and the order of priority of loading of the demands to the assigned depots. Then, efficient constructive heuristics are used to deal with timing constraint satisfaction (i.e. determining a feasible overall schedule according to the assignment performed by the GA) and truck dispatching in the second sub-problem. For the sake of clarity, we describe the GA and the constructive heuristics in separate subsections.

4.1. The genetic algorithm

Genetic algorithms belong to a class of stochastic search methods that work iteratively on a population of candidate solutions of the problem (individuals), performing a search guided by the ‘fitness’ (i.e. the value of the objective function) of each solution. In particular, the higher the fitness, the more the genes of a solution are likely to be propagated to the next iterations. This Darwinian principle is emulated with specific crossover, mutation and selection operators, which are applied with stochastic mechanisms that
make the GA explore solutions with increasing fitness. One of the frequently acknowledged merit of these optimization algorithms is their flexibility with regards to the characteristics of the objective function, as they do not rely on specific a priori hypotheses (e.g. continuity and convexity).

Every GA requires a preliminary definition of an encoding strategy to transform a generic solution of the problem into a string of symbols, chosen from a pre-specified alphabet and suitable to the application of recombination operators for generation of new solutions (i.e. crossover and mutation operators). In GA literature, an encoded solution is generally referred to as chromosome, and a single parameter of the solution vector is called a gene. As mentioned, an extremely rich literature on GA-based sequencing and scheduling methods is available to date, and a considerable variety of different encoding strategies have been devised to address specific requirements. When dealing with scheduling problems, there are several conflicting requirements in the definition of the coding strategy. In particular, the coding strategy should be devised so as to have compact chromosomes that completely characterize the associated solution. Ideally, the coding should be defined so as to ensure that the crossover (mutation) of two (one) chromosome(s) describing legal (i.e. satisfying all the constraints) solutions always lead to legal new solutions. For complex scheduling problems, such as the one considered here, guaranteeing legal offspring involves either extremely long chromosomes (which may compromise the search efficiency of the GA) or very complex and computationally demanding crossover and mutation operators. Many different solutions to overcome this problem in related contexts have been proposed in the literature (e.g. Lee and Choi, 1995; Michalewicz, 1996; Zhang et al., 1997; Ozdamar, 1999, to mention only some), and the main ones have been considered as possible alternatives in a preliminary investigation for algorithm configuration. Due to the size of the typical instances faced by the considered supply chain, none of the available methods was able to provide sufficiently compact and transparent chromosomes for instances of the typical size faced by the supply chain in average workdays (several hundreds of jobs to be assigned, sequenced and routed). In order to cope with problem size and algorithm complexity, we have adopted a hierarchical hybrid approach (corresponding to the two-step approach explained earlier) in which the GA only processes a part of the whole scheduling problem, while constructive heuristics are applied to determine the complete schedule associated with a chromosome each time the fitness of a chromosome is computed. It is also important to mention that the hybridization strategy is also particularly suitable for the considered problem because it makes possible to explicitly handle the additional requirements of the scheduling problem that are not incorporated in the model by appropriate definition of the local constructive heuristics.

To illustrate the basic mechanisms of our strategy, let us first focus on the chromosome encoding. The chromosome is made up by two separate parts, both containing $R$ (the number of demands) elements, as described in Fig. 2 for $R = 6$ (in a real problem $R$ can vary up to hundred or more). The first part defines the assignment of demands (requests) to the depots. Each gene is an integer number between 1 and $D$ (the number of depots): the $l$th gene of this first part of the chromosome indicates the depot to which request $r_l$ is assigned. For instance, in the chromosome represented in Fig. 2, requests $r_1$ and $r_4$ are assigned to depot 1, $r_2$ to depot 3 and all the remaining ones to depot 2. The second part of the chromosome establishes the order in which the $R$ requests will be considered in the construction of the complete schedule of the production chain. The $l$th gene in this second part indicates the demand that will be considered at the $l$th step.

<table>
<thead>
<tr>
<th>[Customer’s] Request-to-Depot Assignment</th>
<th>priority of request in schedule construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>1</td>
</tr>
<tr>
<td>$r_2$</td>
<td>3</td>
</tr>
<tr>
<td>$r_3$</td>
<td>2</td>
</tr>
<tr>
<td>$r_4$</td>
<td>1</td>
</tr>
<tr>
<td>$r_5$</td>
<td>2</td>
</tr>
<tr>
<td>$r_6$</td>
<td>4</td>
</tr>
<tr>
<td>$p_1$</td>
<td>5</td>
</tr>
<tr>
<td>$p_2$</td>
<td>6</td>
</tr>
<tr>
<td>$p_3$</td>
<td>1</td>
</tr>
<tr>
<td>$p_4$</td>
<td>3</td>
</tr>
<tr>
<td>$p_5$</td>
<td>2</td>
</tr>
</tbody>
</table>

Fig. 2. A generic chromosome.
of the scheduling construction procedure. Thus, demand corresponding to \( p_1 \) will be considered first, followed by demand corresponding to \( p_2 \), and so on. For instance, in the chromosome below, request \( r_4 \) (represented by the integer 4) corresponds to \( p_1 \) and will be the first one to be allocated in the scheduling plan. The next demand in the order of priority is \( r_5 \) (represented by integer 5 and corresponding to \( p_2 \)), followed by request \( r_6 \) (corresponding to \( p_3 \)), and then \( r_1 \), \( r_3 \), and finally \( r_2 \). Clearly, the second part of the chromosome can be any permutation of the sequence of integers 1, 2, \ldots, \( R \). If \( R \) is small (e.g. upto 10), one could consider all possible permutations and suffice by using only the first part of the chromosome. However, for realistic \( R \) values, this enumerative approach is simply not feasible due to time and resource constraints, and hence genetic search is also needed here.

In terms of the decision variables, the first part of the chromosome defines the values of \( Y_{ij} \), while assignment variables \( Y_{oi} \) and \( X_{ikm} \) are not specified, but computed by the constructive heuristic procedure (described in the next subsection), which is called every time the fitness of a new chromosome must be computed. The proposed encoding scheme is integer coding (used for assignment purposes) combined with order (or permutation) based encoding for the priority assignment. Both coding methods have been used extensively in the GA literature, and proven to be effective in a large variety of combinatorial problems (see e.g. Michalewicz (1996), or Ishibuchi et al. (2003)). Although a binary coding has some advantages in terms of implicit parallelism of the search, we apply an integer coding to keep the chromosome simple and reduce the overhead of coding/decoding. The coding described above is effective in our problem, as shown in the next sections. It also gives a straightforward description of the part of the solution that can be easily interpreted or manipulated by plant managers (e.g. modifying the assignment of a demand so as to have it produced on a specific depot or scheduled with a higher priority).

The main schema of the GA is summarized in Fig. 3 using a descriptive meta-language. The set \( \text{Pop}(i) \) represents the set of solutions (including the associated fitness) composing the population at \( i \)th iteration (generation). The role of each meta-function used in the description can be summarized as follows.

- **random_pop**: this function creates a random initial population of 100 individuals in our experiments.

```c
/* Single Criterio n GA */
/* Algorithm Startup */
\text{i} = 1;
\text{Pop}(1) = \text{random_pop}
\text{fitness_eval(}\text{Pop}(1))
\text{i} = 2;

/* main loop of the GA */
\text{WHILE terminating condition == false}
    \text{p\_best = findbest(}\text{Pop}(i-1)) /* elitist preservation of the best-known individual*/
    \text{Pop}(i) = \text{select(}\text{Pop}(i-1), sel\_ops);
    \text{Pop}(i) = \text{crossover(}\text{Pop}(i));
    \text{Pop}(i) = \text{mutation(}\text{Pop}(i));
    \text{fitness_eval(}\text{Pop}(i))
    \text{Pop}(i) = \text{pop}(i) \cup \text{p\_best}
    \text{i} = \text{i} + 1;
\text{END WHILE}
```

Fig. 3. The basic structure of the GA.
fitness_eval(\text{Pop}): the fitness of each new individual (i.e. resulting from a random initialization, or from a crossover or mutation operation that has produced a solution differing from its parent(s)) in \text{Pop} is computed. As mentioned, the chromosome of an individual only specifies a part of the decision variables of the scheduling process ($Y_{id}$), while the remaining variables ($Y_{0i}, X_{ikm}$) and the scheduled times for each operation are determined by a sequence of constructive heuristic algorithms that will be described in the next subsection. Once the overall schedule of the whole supply chain has been defined by the constructive algorithms, the value of the cost function associated with each individual is computed and assigned as the fitness of the chromosome.

select(\text{Pop}, \text{sel_ops}): this function returns a new population of solutions selected from those in \text{Pop} with a strategy that assigns higher probability of selection to individuals with higher fitness. We use tournament selection (Michalewicz, 1996) with two individuals for each tournament.

crossover(\text{Pop}): this function randomly selects couples of solutions in \text{Pop} to perform a crossover that returns two new individuals, which partially inherit some characteristics of both parents. After the crossover, the resulting offspring replaces the two parents. Finding effective operators and their optimal rate of application for a given problem is a key-issue for any successful application of a GA. The literature offers an extensive amount of comparative analyses of recombination operators for various assignment, sequencing or scheduling problems. While few conclusions are sufficiently general to be extended to our case study, an exhaustive comparison of all the possible combinations of operators is certainly prohibitive in our context. However, there are recent works (e.g. Nearchou, 2004; Ishibuchi et al., 2003), which suggest efficient operators for either part of the chromosome. Although finding more efficient operators for our coding strategy may be an interesting direction for further research, here we focus only on these known operators.

Our crossover operator randomly selects a chromosome cut point (Fig. 4). If this point falls on the first half of the chromosome, it performs a standard single-cut crossover (Michalewicz, 1996) to the first part of the chromosome (request to depot assignment), otherwise it performs an order-based crossover (Ishibuchi et al., 2003) on the remaining part. Based on a preliminary set of runs for algorithm configuration, the probability of crossover has been chosen equal to 0.5 (i.e. about 50\% of the individuals in \text{Pop} are subject to crossover).

mutation(\text{Pop}): this function randomly alters a solution to obtain a new one. Similarly to crossover, we selectively apply two different mutation operators. A gene in the chromosome is selected randomly. If it belongs to the first part, the gene is replaced by a randomly drawn integer between 1 and $D$. Otherwise, the inversion mutation (two randomly selected genes are swapped in the sequence) is applied to the order-based part. The effects of this operator are illustrated in Fig. 5. Based on preliminary experiments, the probability of mutation was chosen equal to 0.02. This value (as well as the rate of crossover) agrees with the values usually suggested in technical literature on GAs.

terminating condition: for all our experiments, we stopped the algorithm after 200 generations.

<table>
<thead>
<tr>
<th>SINGLE-CUT CROSSOVER</th>
<th>ORDER BASED CROSSOVER</th>
</tr>
</thead>
<tbody>
<tr>
<td>parent string 1</td>
<td>1 3 2 1 2 2</td>
</tr>
<tr>
<td>parent string 2</td>
<td>3 1 2 2 3 1</td>
</tr>
<tr>
<td>new string 1</td>
<td>3 1 2 1 2 2</td>
</tr>
<tr>
<td>new string 2</td>
<td>1 3 2 2 3 1</td>
</tr>
<tr>
<td></td>
<td>1 3 2 1 2 2</td>
</tr>
<tr>
<td></td>
<td>3 1 2 2 3 1</td>
</tr>
<tr>
<td></td>
<td>1 3 2 2 3 1</td>
</tr>
<tr>
<td></td>
<td>1 5 4 2 3 6</td>
</tr>
<tr>
<td></td>
<td>1 5 4 2 3 6</td>
</tr>
<tr>
<td></td>
<td>1 3 2 2 3 1</td>
</tr>
<tr>
<td></td>
<td>1 5 4 2 3 6</td>
</tr>
</tbody>
</table>

Fig. 4. Examples of the crossover operator.
Once a part of the solution is specified in the chromosome, a constructive heuristic procedure (CHP) is used to determine a legal schedule for the entire supply chain. The CHP is composed of two separate parts, respectively dealing with the scheduling of loading operations at the PCs, and the scheduling of job deliveries by trucks. For the sake of brevity, it is not possible to provide an exhaustive description of all the steps performed by each part of the CHP, and so, in the following we focus on the main mechanisms of each part.

4.2. Constructive heuristic procedure: Scheduling loading operations

Basically, the procedure starts to process the demands following the order of priority specified in the second part of the chromosome. The operations of the CHP are relatively simple when there is only one job for each request ($R = N$). If this is the case, the first processed demand is scheduled on the PC assigned in the chromosome, unless the PC-to-customer distance, PC loading rate, and customer unloading rate make this assignment unfeasible. In the latter case, the demand is redirected to the nearest depot to the delivery location of the demand. When no conflict with previously scheduled demands is detected, a demand $r$ is scheduled so that the unloading (of its first job) starts exactly at the $EDT_r$. The second and successive demands are scheduled with similar criteria. Firstly, it is inspected if they can be scheduled at the assigned PC so that they are delivered and unloaded within the specified time-window. When a demand $r$ is scheduled at a PC that has already been assigned other demands, two different situations can occur: (1) the customer-specified time-windows for the requests are such that the production times of $r$ is not in conflict with the previously assigned ones or (2) $r$ is in conflict with some other job loading at the PC, and the assignment cannot be accepted as is. The partial overlap of loading times is one of the simplest conflicts that may arise in the complex scheduling problem addressed here. The CHP firstly tries to shift forward the start of the loading time-window of $r$ until it is no longer overlapping with the others assigned to the same depot. Of course, this operation may have negative effects. In particular, the start time of unloading of $r$ will be shifted forward as well and consequently it may happen that the delivery cannot be completed in the time requested by the customer. In this case the CHP makes a second adjustment trial, this time shifting backward the loading window of $r$ until there is no overlapping. This operation is in general less favorable than the previous one because it implies that the truck will arrive at the delivery location before the $EDT$, and thus it will have to wait. The waiting time cannot be so long that the concrete starts to set before its complete unloading. If this constraint is violated, there is no other solution except for either reassigning the demand to another PC, or making other adjustments that involve also the already assigned demands, i.e. those having a higher priority than the one of $r$. We must note that whenever a request is reassigned to a depot differing from the one specified in the chromosome, the CHP will change the chromosome accordingly. If a request cannot be successfully reassigned to one of the available depots of the company, then it is outsourced. Thus, the priority of demands in schedule construction strongly influences the way the demand are scheduled over time. The way schedules are (re)constructed to meet all the timing constraints also makes possible that two or more different chromosomes lead to the same final schedule, and thus to the same fitness value.
In the most general case, each demand is composed of several jobs. In this case, the CHP has to perform a considerably larger number of operations. The general sequence of operations performed by the CHP is summarized in Fig. 6. Also in this case the CHP examines each demand in the chromosome in the order of assigned priority. Let us focus on a given demand \( r \): after checking for possible infeasibility of the assignment due to excessive distance between delivery location and assigned PC (if it occurs, it is handled as described before), the algorithm starts to examine and schedule each job composing the demand. The algo-

![Diagram of the CHP process]

Fig. 6. The main steps of the CHP and its integration in the GA.
rithm computes the SLT of the first job of the demand based on the customer-specified EDT and of the company-specified UWT, avoiding the overlapping of the loading windows as explained in the previous section. If one of the adjustments leads to a successful schedule, then the gene specifying the assignment of the demand to the PC is confirmed and marked as unchangeable. Otherwise, the algorithm proceeds by attempting to assign the second job of the demand to the PC, using almost the same procedure described above. It is worth noticing that when the CHP operates on the second or subsequent jobs of a demand, it has to verify that both the preceding but not yet assigned jobs, and the following ones can be scheduled satisfying the delivery time constraints. If also the second job cannot be assigned to the depot, the algorithm tries with the subsequent ones, until either one of the jobs is assigned to the depot, or none of the jobs composing the demand can be scheduled on the PC. In the latter case (100% jobs must be reassigned elsewhere), the gene of the chromosome is actually changed and the procedure will start investigating the other PCs in the order of shortest distance from depot to the customer’s site.

After successfully assigning one of the jobs of the demand to a given PC, say \( d \), the CHP proceeds to assign all the remaining jobs to the same PC \( d \), guaranteeing that each job will be delivered so that the end of its unloading coincides with the start of unloading of the following job (see constraint \( c_{12} \)).

In a second step, the CHP reconsiders the unscheduled jobs and tries to place them on other PCs, considered in the order of increasing distance from the customer’s site. Note that it can occur that a job of a demand can be allocated in time earlier than other jobs of the same demand having a smaller index. The CHP will then sort the jobs of the same demand so that they have an increasing index. Moreover, this reorganization of the sequence of jobs evenly redistributes the unloading waiting times of all the involved jobs, thus leading to an improved schedule.

At this point, the main task of the CHP is to handle the jobs still not assigned to any PC. The final attempt to assign a job to a PC is performed as follows. Starting from the first PC in the order of increasing distance to customer’s site, the CHP tries to “force” the insertion of the job at the exact time that guarantees the ideal unloading time. The main property of this last attempt is that the CHP now is allowed to shift-backward also the jobs already scheduled in previous steps (including those having a higher construction priority. This operation is likely to have a significant impact on the overall schedule at the PC, and also to increase the value of the cost function associated with the solution. To evaluate the actual advantages of this insertion, the CHP computes the increment of cost caused by the insertion of the job \( i \) in the schedule. If either the increment exceeds the cost of outsourcing of the job \( i \), or the insertion causes a violation of some feasibility constraint, the insertion is rejected and the CHP tries with the next PC. On the contrary, if the insertion determines a cost increment that is lower than the cost of outsourcing, then the insertion is accepted, and the schedule is modified accordingly.

4.3. Constructive heuristic procedure: Scheduling trucks

Once the assignment solution encoded in the chromosome is converted in a feasible loading sequence for each PC, the fleet of trucks must be assigned to jobs (setting the values of decision variables \( X_{ikm} \)) and routed from PCs to customer sites and vice-versa to pickup and deliver loads. Basically, the truck scheduling must guarantee that a truck assigned to a job is available at the loading dock of the supplying PC at the scheduled loading start time. A heuristic procedure, referred to as Truck Schedule Construction Algorithm (TSCA) is in charge of performing this task. Initially, all the jobs that are marked as directly picked-up by customer-owned trucks are removed from the assignment list. Also the TSCA works in two consecutive and separated phases: first, it assigns the jobs produced at a given PC to the fleet of vehicles already owned by and located at the same PC, and second it searches for vehicles for delivering the remaining unassigned jobs. The main operations of the two phases are summarized in Fig. 7.

To illustrate the allocation procedure, let us firstly define the set of hypothetically available trucks at a depot at time \( t \). This set is composed of the trucks that either have not left their base PC from the beginning
of the working day (hereinafter defined as type 1) or have already completed some transport operations and can return to the PC before time \( t \) (type 2). This distinction is particularly relevant, since when both types of trucks are available, the TSCA always tries to assign those of type 2 first, in order to actually use the minimal amount of trucks for servicing all the requests. If the set of available trucks contains some trucks of type 1 over the whole working day, this clearly indicates that the size of the fleet exceeds the actual requirement. Now let us focus on the assignment mechanisms.

At the beginning of the working day \( t = t_0 \), all the trucks of the PC are idle and ready for operation (all the trucks are of type 1). The TSCA allocates the first jobs produced in the working day to trucks of type 1 until the first truck of type 2 becomes available. From that time on, the TSCA always gives higher priority to trucks of type 2. When multiple trucks of type 2 are available, the TSCA ranks them in order of increasing return time (the time at which they are expected to be back at the base PC) and assigns the last one in the rank (the one with the latest return time) first. Therefore, the truck assignment strategy is also referred in this manuscript as **Shortest (truck) Idle Time (SIT)**, because the truck with the smallest idle time at the PC is the one assigned first. The reason of using such a priority strategy for the trucks is twofold. Firstly, as mentioned earlier, this strategy tends to use a minimal number of trucks in the assignment. Secondly, instead of evenly distributing the idle times among trucks, the SIT strategy causes some trucks to have longer idle times between assigned services. In this way, these trucks can be profitably assigned to jobs.

**Fig. 7.** The truck schedule construction algorithm.
of other PCs, as done in the second part of the TSCA. When a job is assigned to a truck, the variable \( X_{ikm} \) is updated accordingly. When the first assignment strategy is unable to find a truck for a given job, the job is temporarily marked as *undeliverable*, and its assignment is postponed to the second part of the procedure. The first part of the TSCA proceeds with the job-to-truck assignment until it has inspected all the jobs. At this point, either all the jobs have been successfully assigned to the trucks of their respective supplying PCs, or there is a set of unassigned jobs marked as *undeliverable* that still needs to be handled. To sum up, the first phase of the TSCA attempts to assign the jobs of a PC to the smallest number of trucks already located at the PC.

The second phase of the TSCA involves the jobs marked as undeliverable. The main steps of this second part are summarized in the right-hand part of Fig. 7. As mentioned, jobs may be undeliverable because either they are scheduled on a PC that is not equipped with delivery vehicles (and explicitly relies on the support of trucks from other PCs), or the trucks of the PC have already been assigned to other delivery operations. In the first part of this second phase, the sets of unassigned jobs at each PC are merged and sorted in the order of increasing \( SLT \). Let us focus on the first one of the resulting list, say job \( i^o \), and let us call \( d^o \) the PC supplying the job. The TSCA considers the set of remaining trucks sorted by completion time of last operation, and tries to assign \( i^o \) to the first truck in this list, say \( k^o \). To add \( i^o \) to the schedule of \( k^o \), the TSCA inspects various insertion possibilities (either placing it after the last job, or inserting it between two previously assigned jobs). If no insertion meets all the constraints, the procedure considers the next truck in the list. If job \( i^o \) cannot be assigned to any truck in the list of remaining ones, then a request for hiring an external truck is issued. In conclusion, at the end of TSCA, all the decision variables \( (Y_{id}, Y_{oi} \text{ and } X_{ikm}) \) are assigned and the resources are scheduled so as to meet all the problem constraints.

5. The case study

Our research work is based on a supply chain composed of five PCs located in the Netherlands. The fleet of trucks consists of 49 vehicles housed in two PCs. As we can see from Fig. 8, most of the PCs of this company are located around the Rotterdam port area. Though the location of the PCs is strategically planned, note that there are only two base depots for the trucks. This means that three PCs will have to rely on the other two for delivering their produced concrete. Our investigation focuses on information regarding a typical working day, with several requests from various customers spread over a large area surrounding the supply chain. Based on the available data, we generated a further set of 250 hypothetical instances having extremely variable characteristics (number and size of requests, concentration of requests in specific areas, conflicting time constraints, etc.), in order to evaluate the effectiveness of the proposed approach in a wide range of differing operating scenarios. It is worth mentioning that some characteristics derived from the analysis of the available demand patterns are noticeably close to those discussed in related literature (high density of demand between 7:00–9:00 and 13:00–15:00, normal distribution for morning orders, exponential distribution for afternoon orders (Matsatsinis, 2004)). Table 1 summarizes the values in normalized Cost Units (CU) of the main cost parameters, reflecting the actual economical costs associated with the various production activities.

Each considered instance differs in the number and characteristics of the requests. In particular, each demand specifies a delivery time-window \( (EDT, LDT) \), a quantity \( Q \) required (in m\(^3\)), a maximal delivery size \( Mds \), a fixed waiting time \( (Fix) \), an unloading rate \( (UR) \) and a percent of the truck that must be left empty \( (Per) \). The trucks have a maximal capacity \( C_{\text{max}} \) of 10 m\(^3\). In general, customer requests have very narrow time-windows, which imposes to schedule the delivery of the first job very close to the EDT. The average truck speed used in our model is 60 km/h while the concrete setting time \( T_{\text{set}} \) is 150 minutes. The working day for a truck is between 5:00 AM to 4:00 PM, and if some truck is scheduled to return to base location later than the end of the working day, an additional cost is incurred.
The prototype of our GA-based hybrid scheduling strategy is developed in Matlab mathematical programming environment. An execution of a single run of the GA configured as described in the previous section takes approximately 6 minutes on a Pentium 4 2.6 GHz. Although execution times could be dramatically reduced by translating the prototype algorithm into more efficient programming environments (e.g. C code), it is worth noticing that even the average execution time of the Matlab code is short enough to allow a quasi-real-time rescheduling in case a new urgent request is received while the current workplan is already started. Fig. 9 shows the details of the GA convergence. The subfigure (A) shows the average and best fitness in the population of a typical run of the GA, while subfigure (B) shows the standard deviation of the fitness in the same run. Subfigure (C) shows the average convergence of the GA over 20 runs starting from

<p>| | |</p>
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<tbody>
<tr>
<td>CP</td>
<td>10</td>
</tr>
<tr>
<td>CA</td>
<td>15</td>
</tr>
<tr>
<td>PT</td>
<td>2000</td>
</tr>
<tr>
<td>HC</td>
<td>10000</td>
</tr>
<tr>
<td>XTR</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost for each km of travel of the trucks</td>
<td></td>
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<tr>
<td>Penalty for idle time</td>
<td></td>
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<tr>
<td>Cost (loss of income) for m$^3$ of concrete to outsource</td>
<td></td>
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<tr>
<td>Cost of an hired truck</td>
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<tr>
<td>Cost (extra pay) for each minute of working out of the standard working time</td>
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different initial random populations (error bars indicate the standard deviation over the various runs) on a reference instance. The statistical performance of the GA is quite satisfactory. The convergence to the final value is quite regular and conform to the typical behaviors of well-configured GAs, and the variance over different replications is sufficiently small. The performance analysis indicates that after 200 generations, the rate of improvement of the solution becomes significantly slower than in the previous generations. Therefore, the termination condition mentioned in Section 4 is sufficient to ensure the convergence of the GA.

To test the effectiveness of the proposed hybrid GA approach, we compare it with four different scheduling policies obtained by applying assignment criteria that are suggested by the experts as main criteria to build their schedules. Namely, the typical decision criteria in this context assign service priorities based on either the distance of the customer’s site from the nearest PC in the chain, or the size of the requests (higher priority to larger orders). Analogously, also trucks are generally assigned using a constructive procedure that takes into account distances and truck idle times. We use four policies for comparison that are obtained by different combinations of these heuristic rules:

![Fig. 9. Statistics of GA convergence over single and multiple replications.](image)
(a) **SD/SIT (Shortest Distance/Shortest Idle Time):** this heuristic criterion first sorts requests by decreasing size. Then, it starts allocating each job to the PC that is closest to job delivery location. Conflict and timing constraints are handled with adjusting procedures that are analogous to those mentioned in the previous section. When a job cannot be assigned to the nearest PC, the algorithm retries with the next PC in the order of increasing distance from delivery location (shortest distance). If no PC can supply the job, it is outsourced. Once all the jobs are scheduled, they are assigned to trucks. The strategy for truck assignment attempts to load each job on the truck that has been idle for the shortest time (shortest idle time).

(b) **SD/LIT (Shortest Distance/Longest Idle Time):** this is a variant of the previously described policy in which only truck assignment is changed by trying to load each job on the truck that has been idle for the longest time (longest idle time).

(c) **SW/SIT (Smallest Workload/Shortest Idle Time).** After sorting requests by decreasing size, this heuristic algorithm tries to assign each job to PCs considering them in the order of increasing (already assigned) workload (smallest workload). The truck assignment strategy is the same as in case (a).

(d) **SW/LIT (Smallest Workload/Longest Idle Time).** This is a variant of policy (c), in which the truck assignment strategy is changed to **LIT**.

In order to provide a clear idea of the results obtained by the hybrid GA approach, let us focus on the scheduling of the production of a demand pattern observed during a typical working day of the supply chain. Table 2 summarizes the results obtained by the five considered policies. This case considers 71 demands for a total amount of 2116.3 m$^3$ of ready-mixed concrete, divided in 258 jobs. Timing details of each demand are summarized in Fig. 10. It is worth noting that the GA-based policy is able to find a schedule that does not entail outsourced jobs, while also minimizing the number of hired trucks necessary to deliver the concrete to customers. The total cost of the solution obtained by the GA is about 20% lower than the one provided by SD/SIT, which is the most effective one amongst the heuristics compared with the GA. In particular, being focused on the optimization of truck routes, the SD/SIT is able to provide the smallest cost associated to transportation, at the expense of longer overall amount of waiting times. It should be remarked that the large amount of waiting times is not evenly distributed between the operations (Gantt charts are omitted for brevity), so the solution found by SD/SIT is not significantly more delay-tolerant than the one obtained with the GA. On the contrary, the job distribution obtained with the GA provides a considerably increased overall length of truck routes, which is fully compensated by the ability to assign all the requests to the five PCs of the supply chain.

The Gantt charts of loading and transportation operations corresponding to the solution obtained with the GA are shown in Figs. 11–13.

<table>
<thead>
<tr>
<th>Table 2</th>
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<tr>
<td>Summary of the results on a reference instance</td>
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<table>
<thead>
<tr>
<th>Components</th>
<th>SD/SIT</th>
<th>SD/LIT</th>
<th>SW/SIT</th>
<th>SW/LIT</th>
<th>GA</th>
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<tr>
<td>Jobs outsourced</td>
<td>3 (28 m$^3$)</td>
<td>56 000</td>
<td>3 (28 m$^3$)</td>
<td>56 000</td>
<td>6 (48 m$^3$)</td>
</tr>
<tr>
<td>Hired trucks</td>
<td>17</td>
<td>170 000</td>
<td>22</td>
<td>220 000</td>
<td>21</td>
</tr>
<tr>
<td>Extra pay (minutes)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td>Empty trips (km)</td>
<td>5290</td>
<td>52 900</td>
<td>5119</td>
<td>51 190</td>
<td>7123</td>
</tr>
<tr>
<td>Loaded trips (km)</td>
<td>3129</td>
<td>31 290</td>
<td>3129</td>
<td>31 290</td>
<td>6716</td>
</tr>
<tr>
<td>Waiting time (minutes)</td>
<td>7214</td>
<td>108 210</td>
<td>15 315</td>
<td>229 725</td>
<td>4803</td>
</tr>
<tr>
<td>Total cost</td>
<td>417 952 CU</td>
<td>587 757 CU</td>
<td>515 856 CU</td>
<td>624 856 CU</td>
<td>325 720 CU</td>
</tr>
</tbody>
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Almost one third of the requests (including most of the larger ones) have their EDT between the 7:00 and the 9:00 AM (Fig. 10). This high concentration of demands requires that all the PCs contribute to the production. In this time interval, the effort of all the PCs is clearly visible in Fig. 11, which reports the Gantt chart of the loading operations at the five PCs. It can be noted that between 6:30 and 9:00 loading operations are continuously performed without pauses. Even the Tilburg-PC3 (the less favorable due to its peripheral position) is not allowed to remain idle in the first part of the day. It can be noted that PC3 has to start mixing earlier than the other ones, due to its larger distance from most of the customers. The concentration of such a large number of deliveries in a relatively short time-window implies several noticeable effects. Firstly, many concrete batches have to be mixed considerably earlier than the optimal time, and consequently the trucks delivering these jobs may arrive greatly in advance at the customers’ sites. This effect can be noted in Fig. 12 observing the waiting times (depicted in white) before the first task of each truck (Fig. 13 reports a detail of the Gantt chart to better illustrate the sequence of operations scheduled on each truck). Secondly, a great number of trucks is required in this part of the day. In fact, it can be noted in Fig. 12 that many of them are used only until 9:00 AM. After this time, about half of the fleet returns to the relative base location as the supply operations become less critical, as also visible in Fig. 11, where some relatively short idle times between loading operations are allowed in the central part of the morning. In particular, the schedule found by the hybrid GA tends to concentrate these idle times on the Tilburg-PC3, due to the aforementioned distance of this center from most customers’ sites. As shown in Fig. 10, a second peak of demands occurs at about 1:00 PM, causing four PCs to reenter the uninterrupted loading stages. Finally, after 3:00 PM there are no more demands and the trucks are allowed to return to their base depots.
Let us now consider the performance evaluation over the whole set of considered (hypothetical and real world) problem instances. The 250 different demand patterns have been grouped in five classes of gradually increasing complexity (labeled from very low, low, average, high, and very high difficulty) based on the average number of hired trucks, and on the average quantity of outsourced production resulting in the solutions found by the four heuristic strategies used for comparisons. These performance indices were chosen to obtain a fairly realistic estimation of the difficulty of the instances, which is not only related to the number and size of the demands, but also strongly affected by the interference between various orders, the customer-specified time-windows, unloading rates and additional requirements.

The result of the experimental investigation is summarized in Table 3. Since in every considered instance the GA-based hybrid approach significantly outperforms all the terms of comparisons, the table reports the average percentile increment of the cost function provided by each heuristic with respect to the average result obtained by running the GA 10 times for each instance.

It can be noted that in all the considered cases, the SD/SIT is the best strategy among the methods used for comparison, with an average loss with respect to GA of about 15% in the easiest instances. In fact, when all the PCs mix at a rate that is significantly lower than their maximum capacity, good overall solutions can be easily found by assigning the requests to the nearest PCs. The discrepancy of cost values raises up to nearly 50% in the cases of moderate and high complexity, owing to the optimized distribution of the loads performed by the GA. The reduced difference of costs in the case of very high complexity is due to the fact that in these cases the overall demand exceeds the maximum productive capacity of the supply chain. Thus, also the solutions found by the GA entail a significant amount of outsourced production, with associated additional costs that also flatten the differences between the most effective scheduling policies and the less

Fig. 11. Chart of loading operations at the PCs.
performing ones. It should also be pointed out that, although the experimental investigation was carried on using the cost weights (derived from industrial practice) summarized in Table 1, the relative performance of the compared algorithms does not depend significantly on the choice of these weights. In fact, in several tests, we observed that applying a variation to a cost weight leads the GA to converge to a different solution, whose overall fitness is still significantly superior to that of all the compared heuristics.

Finally, we carried out an investigation regarding the robustness of the found solutions to stochastic perturbations, such as transportation delays due to traffic or other unexpected events. This analysis is obtained with the aid of a discrete-event simulation of a detailed model of the supply chain, developed within the Rockwell Arena 7 discrete event simulation environment. In the simulated scenarios, the actual speed of trucks is modeled with a triangular distribution. In particular, while the median value of the distribution is set equal to the truck speed assigned in the deterministic model, the left- and right-hand half-widths of the distribution are progressively enlarged so as to investigate the effects of transportation delays of increasing size.

As mentioned, the tolerance to delays of a given schedule is determined by the amount of truck waiting times $LWT$ and $UWT$ at PCs and at customer’s locations, respectively. For instance, if a truck arrives 10 minutes late at a delivery location, but it was initially scheduled to reach the site fifteen minutes earlier than the customer-specified unloading time (i.e. $UWT = 15$ minutes), the delay is compensated by the waiting time without having any further consequence on the remaining scheduling plan. On the contrary, if a truck returns to a PC later than the scheduled loading time of its next job (i.e. the delay exceeds the planned $LWT$), the perturbation may determine chained delays that may significantly affect all the successive operations. For this reason, the safety parameter $MWT$ defines the lower bound for all the loading and unload-
ing waiting times. It is worth mentioning that higher values for the MWT will certainly lead to more delay-tolerant solutions, but it will also increase the value of the component of the cost function associated with waiting times.

The evaluation of tolerance to perturbations is carried on a reference instance selected from the class of hard-difficulty. We consider 12 discrete increments for the half-width of truck speed distribution (up to 30 km/h, which would cause on some long routes delays of more than two hours), and 6 increasing values for the safety parameter $MWT$. For each combination of half-width and MWT, 20 different replications of the discrete-event simulation of the supply chain are run. The final results are summarized in Table 4, which reports the percentile number of replications in which at least one of the following unacceptable events occurred:

![Fig. 13. Detail of the trucks Gantt diagram.](image-url)
• A truck arrives late at a customer site, either violating the continuity of the unloading process or exceeding the concrete setting time.
• A truck returns late to the PC for its next job, delaying the next loading operations at the PC.

Table 4 shows that solutions capable to fully tolerate even reasonably high variations of average truck speed can be found by appropriately setting the value of the safety parameter MWT. The costs associated with solutions with increasing values of the MWT are summarized in Table 5. It should be noted that the extension of the safety margin does not only affect the cost associated with waiting times, but obviously also involves cost related to external truck hiring, since the utilization of the internal fleet is significantly reduced by the increased waiting times. The results in Table 5 indicate that both the overall costs and the number of hired trucks have an approximately linear growth with the safety factor MWT. This particular feature of the proposed model makes it possible to easily determine in advance the value of MWT that provides the desired tradeoff between delay tolerance and final cost associated with the solution found by the proposed GA-based scheduling strategy.

6. Conclusions

In this work, we considered the problem of finding an optimized schedule for the just-in-time production and delivery of ready-mixed concrete on a set of distributed and coordinated production centers. Our attention was firstly focused on the development of a complete and detailed deterministic model of the considered supply chain, incorporating all the specifics that make it considerably different from other formulations of similar scheduling and routing problems. In a subsequent step, we described an effective scheduling algorithm based on the proposed model. The scheduling algorithm combines a GA and a set of constructive heuristics, which guarantee the determination of a feasible schedule for any given set of
requests. The proposed scheduling algorithm was compared with other four constructive heuristics on an industrial case study using a comprehensive set of problem instances. The results obtained illustrate the interesting potential of the proposed approach. Firstly, in the solutions found by the GA the amount of requests that are redirected to external companies, or that need hired trucks for their delivery, is in general very small compared to the other scheduling strategies. Secondly, the proposed model allows the definition of safety margins for minimizing the effects of transportation delays. With the aid of a discrete-event simulation campaign, we have shown that schedules capable of tolerating considerable variations of truck average speed can be found with the proposed algorithm. However, the proposed solution is based on a sequential determination of two sets of relevant decision variables. While this reduces the complexity of the problem, the global optimality of the final solutions cannot be guaranteed.

Our research work is rich of promising directions for further investigation. Even if the proposed GA is able to find satisfactory solutions in short execution times, such an optimization algorithm can be refined in a number of different ways, e.g. devising more efficient crossover and mutation operators. Furthermore, the proposed GA considers the optimization of a single cost function. In practice, the planners consider multiple objectives, and thus multi-objective optimization with evolutionary algorithms (see e.g. Tan et al. (2001, 2003)) is a promising direction to follow. At the moment, a multi-objective version of the proposed algorithm, which is capable of finding the Pareto front of non-dominated solutions with respect to the single components of the cost function (distances, cost of outsourcing, waiting times/safety margins) is under development. Finally, long term research involves the investigation of innovative paradigms based on distributed optimization, in which enhanced reactivity and fault tolerances are achieved by distributing the scheduling task between various decision nodes located at each PC of the supply chain, instead of concentrating it to a centralized optimization engine.

Acknowledgements

The authors wish to thank the Special Issue Editor prof. K.C. Tan and the anonymous referees for their valuable suggestions.

Appendix A. Definition of symbols and acronyms

A. demand related

$r \in \{1, \ldots, R\} \quad \text{customer or demand-related index. } R \text{ is the number of customers/requests processed in the considered time horizon}$

$Q_r \quad \text{quantity of cement requested in } r\text{th demand}$

$[EDT_r, LDT_r] \quad \text{customer-specified earliest and latest delivery time for request } r$

$Per_r \quad \text{user-specified percentage of truck capacity that should not be used}$

$Mds_r \quad \text{maximum size of a deliver allowed by customer } r \text{ to a single truck}$

$Fix_r \quad \text{user-specified fixed waiting time at the destination}$

$UR_r \quad \text{rate of unloading of customer } r$

$Tset_r \quad \text{setting time of the type of concrete requested by customer } r$

$Z_r \quad \text{number of sub-deliveries in which a request exceeding truck's capacity is divided. A sub-delivery is referred to as a } \text{job. } \text{We assume that all the sub-deliveries to the same customer have the same size. }$

In particular, when a request $r$ can be handled by a single truck $Z_r$ is equal to 1, otherwise it is computed taking into account the user-specified limitations on the maximum size of delivery $Mds_r$ as follows:
\[
Z_r = \left[ \frac{Q_r}{\min\{C_{\text{max}} \cdot (1 - Per_r), M_{\text{ds}}r\}} \right],
\]

where \(C_{\text{max}}\) is the capacity of a single truck and \([\cdot]\) indicates rounding to the nearest higher integer.

**B. Sub-demand (job) related**

\(i \in \{1, \ldots, N\}\) job-related index relative to the job. \(N\) is total number of jobs to perform. Clearly, it holds that

\[
N = \sum_{r=1}^{R} Z_r.
\]

The sub-demands are arranged in sequential orders, so that the index \(i\) can be interpreted as follows:

\[
i \in \left\{ \underbrace{1, \ldots, Z_1}_{r=1}, \underbrace{Z_1 + 1, \ldots, Z_1 + Z_2}_{r=2}, \ldots, \underbrace{Z_1 + Z_2 + 1, \ldots, Z_1 + Z_2 + \ldots + \sum_{r=1}^{R-1} Z_r}_{r=R}, \ldots, \underbrace{\sum_{r=1}^{R-1} Z_r + 1, \ldots, N}_{r=R} \right\}
\]

\(f_r\) first job of request \(r\)

\(l_r\) last job of request \(r\). According to this notation, there is a biunivocal correspondence between \(r\) and \(i\). For brevity, we denote with \(r_i\) the demand to which job \(i\) belongs

- \(LT_i\) job loading time
- \(SDT_i\) source to destination traveling time for job \(i\)
- \(DST_{ij}\) traveling time between the destination of job \(i\) and the source of job \(j\)
- \(SLT_i\) earliest loading start time that guarantees the completion of the supply (end of unloading) before the concrete sets
- \(LLT_i\) latest loading start time that guarantees the completion of the overall delivery within the expiration of the customer-specified time \(LDT_r\)
- \(ELT_i\) latest loading end time that guarantees the completion of the overall delivery within the expiration of the customer-specified time \(LDT_r\); clearly, it holds that \(ELT_i = LLT_i + LT_i\)
- \(UWT_i\) waiting time before starting to unload at destination of job \(i\)
- \(UT_i\) job unloading time: as we assume that all the jobs composing a single demand have equal size, the unloading time depends only on the size of the demand \(Q_r\) and on the unloading rate of the customer \(UR_r\):

\[
UWT_i = \frac{Q_r}{Z_r} \cdot \frac{1}{UR_r}.
\]

**C. Depot related**

\(d \in \{1, \ldots, D\}\) depot-related index. \(D\) is the number of depots

- \(A(x, \beta)\) distance between two known locations \(x\) and \(\beta\) (either a depot or a customer site)

- \(LR_d\) loading rate at depot \(d\)

- \(FLT_d\) fixed loading time at depot \(d\)

- \(\Gamma_d\) subset of jobs that have their source in the depot \(d\)

**D. Truck related**

\(k \in \{1, \ldots, K\}\) truck-related index. \(K\) is the total number of trucks \((K = K_c + K_o, \text{where } K_c \text{ is the number of trucks of the company, and } K_o \text{ is the number of additionally hired trucks})\)

- \(C_{\text{max}}\) maximum capacity of a truck

- \(V\) average speed of the trucks
**STWD**$_k$ starting time of the working day for truck $k$

**ENDWD**$_k$ ending time of the working day for truck $k$

**DP**$_k$ base location (depot) of truck $k$, $DP_k \in \{1,\ldots,D\} \cup \{H\}$, where $H$ is the base location for hired trucks

**DF**$_k$ depot where the truck $k$ has to start its first load. This variable is introduced because some trucks (e.g. all the hired trucks), may need to move to a depot different from their base location to pick up their first load

**DL**$_k$ last customer served by truck $k$ before its return to base location

**$T^\text{start}_k$** time of departure of the truck $k$ from its base location

**E. Task related**

$m \in \{1,\ldots,M_k\}$ task-related index. A task of a truck is the deliver of a job to its destination. $M_k$ is the maximum number of tasks allowed to a single truck $k$

\[
M_k \leq M = \left\lfloor \frac{\text{length of the working day}}{\text{minimal length of a task}} \right\rfloor
\]  

\[
LWT_{km}\text{ waiting time for loading the } m\text{th task of } k\text{th truck}
\]

**F. Decision variables**

\[
X_{ikm} \in \{0,1\} \text{ if the job } i \text{ is assigned to truck } k \text{ as } m\text{th task, } X_{ikm} = 1, \text{ otherwise } X_{ikm} = 0
\]

\[
Y_{id} \in \{0,1\} \text{ if job } i \text{ is produced at the depot } d, Y_{id} = 1 \text{ and } i \in \Gamma_d \subseteq \{1,\ldots,R\}, \text{ otherwise } Y_{id} = 0
\]

\[
Y_{oi} \in \{0,1\} \text{ if the production of job } i \text{ is outsourced, } Y_{oi} = 1, \text{ otherwise } Y_{oi} = 0
\]

**G. Cost parameters**

\[
CP \text{ cost for each minute of travel of a single truck (independently of truck type, or travel condition (loaded, empty))}
\]

\[
PT \text{ loss per m}^3 \text{ of outsourced product}
\]

\[
CA \text{ penalty for waiting time}
\]

\[
HC \text{ cost per day of an hired truck}
\]

**H. Safety parameters**

\[
MWT \text{ minimal waiting time for a truck before (1) loading its next job at a PC or (2) unloading the concrete at the customer’s site. This parameter is used as safety margin to tolerate transportation delays}
\]

**References**


