Generating periodic forces with the pendulum actuator

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Abstract
This paper presents the construction, modeling and properties of a compact, efficient electro-mechanical device called a ‘pendulum actuator’, used for generating periodic dynamic forces. Essentially, the device is composed of a transducer based on a smart material (e.g., a piezoelectric stack or a magnetostrictive rod), placed in a mechanical structure dedicated to hosting the electric or magnetic circuit for transducer excitation, and the mechanical displacement amplifier. This type of actuator can potentially cover a wide range of applications, but it also presents some fundamental challenges. In particular, generating periodic force signals demands special provisions due to the inherent nonlinear kinematics of the actuator device. This paper illustrates the potential and the limits of the device by developing an accurate mathematical model and presenting an extensive experimental investigation.

Keywords
Force generation, non-linear dynamic modelling, vibration control

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1. Introduction
Unconventional actuators (Janocha, 2004, 2007) are electro-mechanical devices combining smart materials with complex mechanical structures devised for various purposes. Here, the term ‘smart’ refers to materials in which some mechanical property such as shape, stiffness or viscosity can be changed by means of an external electric or magnetic stimulus. Piezoelectric ceramics and magnetostrictive materials are among the most widely used types of smart material, but several other alternatives, such as magnetic shape memory (MSM) alloys (Gauthier et al., 2008) or electro-active polymers (EAP) (Hackl et al., 2005), are attracting the attention of the research community. The typical nonlinear behavior of smart materials combined with challenges in the design of compact, light and efficient mechanical structures make the development of this type of actuator particularly demanding. For these reasons, a large amount of research has been devoted over the past two decades to issues such as design optimization (Li et al., 2002), efficient mechanical amplification (Du et al., 2000), hysteresis modeling and compensation (Iyer et al., 2005; Janocha et al., 2006; Kuhnen, 2003) as well as feedforward and feedback control (Tan and Baras, 2005; Wu and Zou, 2007), to cite only a few.

Force generation is one of the typical domains in which smart actuators exhibit interesting potentialities. In particular, generating periodic forces efficiently within some predefined frequency bands is a typical goal of devices developed for active vibration damping. Dynamic forces can be generated in numerous technical applications by means of an auxiliary mass \( m_2 \) which is connected in an appropriate way with the structural mass \( m_1 \) (see Figure 1). If the motion of the auxiliary mass is controllable, a desired shape of the force signal \( F_2 \) introduced to the structural mass can be applied and stabilized with sensory feedback and control.

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In general, one speaks of a force generator, but if applied for example to the task of vibration damping through compensation of the disturbance force $F_d$, then the device can be described as an active auxiliary mass damper. The dynamic behavior of the force generator depends on its topology as laid out for any given application. If for example a broadband vibration disturbance is to be compensated, then normally an auxiliary mass damper will be implemented that preferably exhibits linear system behavior.

The use of smart materials in active mass dampers requires the development of suitable mechanical displacement amplification due to the limited elongation that can be achieved by the active element. A recent example of this type of actuator is the magnetostrictive auxiliary mass damper (AMD) described in May et al. (2003) and (2006), with which broadband force signals can be generated for reducing structural vibrations. Other applications, however, may demand narrowband dynamic forces. This is the case in helicopters where a strong fundamental disturbance corresponding to the blade pass frequency excites undesired structural vibrations. An auxiliary mass damper with a pronounced resonant behavior is desired; linear, broadband system behavior is not necessary in this case. The device analyzed in this paper is conceived to achieve these purposes. Namely, we consider a smart actuator that is a direct evolution of the ideas developed for the AMD – a ‘pendulum actuator’ (PA). The PA, which was first introduced by May and Janocha (2007), is a device in which the parametric configuration of the kinematic displacement amplification used in the AMD is chosen such as to enable pendulating motion around the only possible equilibrium point, which is simultaneously a singularity point for the device. This peculiarity makes it possible to generate dynamic forces within a narrow frequency band with remarkable efficiency relative to the auxiliary mass but at the cost of a much more complex dynamic behavior, demanding the additional theoretical and empirical investigations to be illustrated in this paper.

The PA is capable of tapping broad application potential, depending on whether it is operated passively, in feedforward or in feedback control (the interested reader is referred to May et al. (2008) and Kuhnen et al. (2006) for details). Depending on the working mode, the range of operating frequencies, the stiffness and the damping behavior can be influenced or controlled. Furthermore, it is possible to achieve self- adaptive behavior with respect to the operating frequency.

It was experimentally shown in May and Janocha (2008) that the passive system behavior of the pendulum actuator exhibits significant vibration damping under conditions of anti-resonance. Adjustment of the magnetic bias current or bias voltage in the driving signal enables the anti-resonant frequency to be varied over a wide range. Vibration attenuation was increased to nearly $-19\,\text{dB}$ by actively driving the pendulum actuator with a manually tuned sinusoidal signal.

While a particular case of sinusoidal force generation using the PA was at the focus of the work presented in May et al. (2009), this scientific contribution presents the detailed derivation of a mathematical model of the PA whose analysis and simulation leads to a description of the PA’s potential and limits for generating periodic forces. An extensive experimental investigation validates the accuracy of the proposed model. Jointly considering the theoretical analysis of the operating behavior on the basis of such a model and the empirical investigation shows optimization potential and thus supports the targeted further development of the design for use in new applications.

The remainder of the paper is organized as follows. First, a mathematical model of the pendulum actuator will be developed in section 2. Sections 3 and 4 are devoted to the analysis of force generation capability and the associated distortion in the frequency domain, respectively. Then, section 5 considers some physical limits in the device and gives perspective regarding the range of its controllability. An outlook and open research issues conclude the paper in section 6.

2. Mathematical model

As shown in Figures 2(a) and 2(b), the transducer element in the pendulum actuator based on smart or active material, in this case a piezoelectric multilayer actuator, is placed under mechanical preload between the two backing plates, which ideally are infinitely stiff in bending. In this way, the entire displacement generated by the piezo transducer equals the relative motion of the backing plates. This opposing motion of the backing plates, which is parallel to the operating axis of the piezo element, is transformed via the kinematic arrangement of the elastic suspension arms into a motion quasi perpendicular to the piezo axis and parallel to the central mounting frame to which the suspension arms are attached. Thus, the dynamic auxiliary mass encompasses the backing plates, the mechanical preload...
mechanism and the piezo transducer and in the present example comprises about 90% of the overall device mass. The acceleration forces associated with the motion of the auxiliary mass parallel to the mounting frame are transmitted to the frame via the elastic suspension arms.

This pendulum device differs from the AMD presented in May et al. (2003) and (2006) in that the angle of the elastic suspension arms with respect to the operating axis of the piezoelectric transducer oscillates between positive and negative values, i.e. the auxiliary mass pendulates about the neutral position corresponding to the said angle being equal to zero. This pendulating motion has several consequences with respect to the performance of the device: (1) for a given displacement of the piezo transducer, the amplitude of displacement of the auxiliary mass and thus of the resulting dynamic force is more than double that which is reached in the AMD; (2) the frequency of the dynamic force signal corresponding to pendulation is half that of the signal used to drive the active material; (3) the relationship between the driving signal and the resulting force is non-linear, exhibiting harmonic behavior that is at the focus of study in this paper. The interested reader is referred to May and Janocha (2007) for a more detailed development of the pendulum actuator concept.

For deriving a mathematical model of the actuator, a schematic illustration of the pendulum actuator and equivalent circuits of its passive and active mechanical components are used (see Figure 3).

Figure 3(a) describes the overall device showing the significant internal forces. Mass $m_1$ represents the mass

![Figure 2. Pendulum actuator. a) Photo of demonstration device with piezo transducer; b) 3D model representation.](image)

![Figure 3. Pendulum actuator. a) Kinematics of the structure; b) and c) equivalent rheological models of the mechanical components.](image)
of the central frame together with the structure upon which the device is acting by means of force $F_2$ and is considered to have no connection to any other structural elements, i.e. floating in free space. Furthermore, the effects of gravity are ignored. $F_2$ represents a disturbance force acting on the same structure. The actuator force $F_2$ is generated by the two masses $m_2/2$, which together comprise the total dynamic auxiliary mass pendulating about the equilibrium point at $\alpha = 0$, and is transmitted to the structure via the elastic suspension arms. $F_1$ is the tensile force in the suspension arms. The so-called pendulum force $F$, which comprises a passive component $F_p$, and an active component $F_A$, acts simultaneously on the left-hand and right-hand auxiliary mass halves (see also Figure 3(b)). The active force component originates for example from a piezoelectric or magnetostrictive transducer. These kinds of solid-state active materials can be modeled in several ways, starting with the usage of simplified linear models and increasing their complexity to take into consideration their nonlinear behavior. Since several effective strategies to pre-compensate the nonlinearities of the active material by preprocessing the excitation signal (voltage or current) are available to date (e.g., Kuhnen, 2003; Tan and Baras, 2005), in this work, a simple linear model is used and is discussed later on in the paper.

The force $F_{py}$ along the $y$-axis results from the passive design elements connecting the auxiliary masses $m_2/2$ with the central mounting frame. In the case of the pendulum actuator depicted in Figure 2(a), these passive elements represent the elastic and damping properties $c_y$ and $d_y$ of the elastic suspension, whose kinematic properties are modeled by rigid lever arms in Figure 3(a).

In order to obtain the mechanical model, both the Lagrangian and direct Newtonian approaches have been used. For brevity, only the Newtonian formulation is shown and discussed briefly here (details on both mathematical derivations of both formulations can be found in Grasso (2008). Considering the forces acting on mass $m_2$, according to the second Newton law, one can write:

$$\begin{cases} \frac{m_2}{2} \ddot{x} + d_x \dot{x} + c_x \Delta x - F_A = - F_1 \cos \alpha \\ \frac{m_2}{2} \ddot{y} + d_y \dot{y} + c_y \Delta y = F_1 \sin \alpha \end{cases},$$

where, based on the kinematics of the device, one can define

$$\begin{align*}
\Delta x &= l \cos \alpha \\
\Delta y &= -l \sin \alpha.
\end{align*}$$

In these equations, $x$ and $y$ represent the horizontal and vertical distances of the mass $m_2$ from the central frame reference, while $s$ is the distance between the backing plates. Furthermore, the force $F_e$ represents the force originating from the structure and acting on the mounting frame, and $F_A$ is the active force generated by the active material inside of the device. It is interesting to note that the equations in (1) can be considered linear differential equations, in case we neglect the term coming from the elastic suspension forces. These terms, in fact, act in the equations according to the geometry of the system, which is not linear.

The acceleration of mass $m_2$, expressed with reference to the Cartesian system with its origin at $x_0,y_0$, can be rewritten using Newton’s law applied to the central frame as:

$$\dot{a}_2 = a_1 + \dot{y} \Rightarrow a_1 = \frac{F_c}{m_1 + m_2} + \frac{m_1}{m_1 + m_2} \dot{y}.$$

Choosing the angular position and the related velocity as state variables:

$$\dot{x} = \left( \begin{array}{c} x \\ \dot{\alpha} \end{array} \right),$$

and substituting the formulas in (2) and (3) into equation (1), one can arrive at the mathematical expression of the state model

$$\ddot{x} = \left( \begin{array}{c} \dot{x} \\ \dot{\alpha} \end{array} \right) = \left( \begin{array}{c} a_1 \\ \alpha \end{array} \right) = \left( \begin{array}{c} K_1(\alpha) \dot{x}^2 + K_2(\alpha) \dot{\alpha} + K_3(\alpha) F_e + K_4(\alpha) F_A + K_5(\alpha) \\ \alpha \end{array} \right),$$

where the time dependence of the variables has been ignored and the following expressions have been defined.

$$\begin{align*}
K_1(\alpha) &= -m_2 \sin \alpha \cos \alpha \\
K_2(\alpha) &= -2(d_2 \sin^2 \alpha + d_y \cos^2 \alpha) (m_1 + m_2) \\
K_3(\alpha) &= m_2 (m_1 + m_2) \sin^2 \alpha \\
K_4(\alpha) &= -2(m_1 + m_2) \sin \alpha \\
K_5(\alpha) &= -2(m_1 + m_2) \sin \alpha (c_y \cos \alpha + 2c_x (1 - \cos \alpha)).
\end{align*}$$

These state equations (5) allow simulating the entire kinematic and dynamic behavior of the device, where
the generated active force $F_A$ and the disturbance force $F_e$ can be considered as inputs. This set of equations describes entirely the device’s behavior, independently of the model used to describe the active material generating the active force. In this work the active material is considered to be a piezoelectric stack actuator, whose behavior can be expressed using the simplified, linearized equation

$$F_A = c_A d V_A - c_A s_A - d_A s, \quad (7)$$

where $d$ is a piezoelectric displacement constant, $c_A$ is the stiffness of the piezoelectric material and $V_A$ is the applied voltage. As in any practical application, the useful range of applied voltage is bounded by saturation behavior, here between 0 V and 150 V for the implemented piezoelectric element. Table 1 summarizes the values of the model parameters used to obtain the simulation results shown in Figures 4, 5 and 14.

The overall model is strongly nonlinear, in fact all of the terms in the state equations (5) exhibit a nonlinear dependence on the state variables. The generated force $F_2$ is also a nonlinear function of the state variables and can be formulated as

$$F_2 = m_2 \sin(\alpha)^2 - m_2 \cos(\alpha) \ddot{\alpha}. \quad (8)$$

It is now of interest to analyze the equilibrium points of the mathematical model, which correspond with the solutions of the set of equations

$$\begin{cases} \dot{\alpha} = 0 \\ \ddot{\alpha} = 0. \end{cases} \quad (9)$$

In this case, the only solution is $\alpha_{eq} = 0$, which corresponds with the initial position. A free response of the pendulum actuator to an initial condition $\alpha \neq 0$ is shown in the state-space trajectory of Figure 4. The trajectory shows sub-critically damped motion corresponding to the experienced behavior of the pendulum actuator. The stability of the equilibrium point can be proven with the help of Lyapunov theory. This is in fact a nodal sink.

<table>
<thead>
<tr>
<th>Table 1. Parameter values used in simulation</th>
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<tr>
<td>Equivalent structural mass $m_1$</td>
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<tr>
<td>Auxiliary mass $m_2$</td>
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<tr>
<td>Elastic suspension length $l$</td>
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<tr>
<td>Passive stiffness $c_A$</td>
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<tr>
<td>Stiffness of longitudinal preloaded spring $c_x$</td>
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<tr>
<td>Longitudinal damping factor $d_x$</td>
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<tr>
<td>Stiffness of vertical preloaded spring $c_y$</td>
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<tr>
<td>Vertical damping factor $d_y$</td>
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<tr>
<td>Piezoelectric constant $d_0$</td>
</tr>
<tr>
<td>Initial suspension angle $\alpha_0$</td>
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</tbody>
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**Figure 4.** Typical free response of the device moving toward the equilibrium point within the state plane.
The pendulum actuator, moreover, is uncontrollable when $\alpha = 0$, since the pendulum force $F$ acting against the two halves of the auxiliary mass $m_2/2$ then has no effect on their vertical motion. This singularity condition is also recognizable from

$$K_4(0) = 0,$$

through which the driving voltage $V_d$ becomes ineffective. Considering the fact that the condition $\alpha = 0$ is both the only equilibrium point as well as a singularity, the pendulum actuator theoretically cannot begin working without an external disturbance that excites a change in the initial condition. In practice, various strategies exist to cope with this problem successfully, taking advantage of imperfections and tolerances in the construction of the device, which cause deviation from the pure mathematical representation. It is important to remark on the existence of such imperfections, because they will be recalled in further considerations.

3. Force generation characteristics

The pendulum actuator device is a narrowband force generator. Its peculiarity is in fact to generate higher forces, even if distributed over a small range of frequencies. This device has resulted from the continued development of the AMD device (various examples have been realized at the LPA, Saarland University), which allows instead the generation of smaller forces but over broader frequency ranges. The following investigations into the force generation capability of the pendulum actuator are carried out for the case of comparatively high base structure mass $m_1$. Consequently, the structure to which the device is attached will experience little motion and the pendulum actuator will be operating near its maximum force generating capability.

Figure 5 shows the generated force $F_2$ resulting from a simulation involving a time sweep of the driving voltage frequency.

Experimental results, which are obtained using the piezoelectric pendulum actuator shown in Figure 2a, confirm this behavior. The experimental device has an auxiliary mass $m_2$ of approximately 0.075 kg, while the length of the elastic suspension $l$ is approximately 18.5 mm, and, also in this case, $\alpha_0 = 0$. The remaining parameters were not precisely identified for the investigations reported here, since the key objective of the experiments was to investigate the force generation capabilities of the device, mainly by varying the driving signal bias, amplitude and harmonic content. All of the force measurements shown in Figures 6 through 13 were obtained from a load cell mounted between the piezo pendulum and a large, stiff laboratory table representing an infinite mass $m_1$. Moreover, the force spectra were obtained from a highly sensitive audio analyzer and the driving signal generated by a high-power voltage amplifier designed to drive piezoelectric devices.

Interestingly, the frequency range of force generation can be modified by acting on the static component (preload) $F_0$ of the so-called pendulum force $F$ existing between the backing plates. In fact, it is possible to modify this parameter of the device by acting on a screw located on the backing plates, or by adjusting the voltage bias. This has the consequence of shifting the generated force frequency range as well as affecting its amplitude. In particular, an increase in the preload force will shift the frequency range to higher frequencies. Figure 6 shows the experimentally recorded shift in the generated force amplitude over the frequency for different voltage offsets keeping constant driving signal amplitude.

![Figure 5. Typical force result obtained by sweeping the driving signal frequency.](image-url)
Another characteristic is discovered when changing the amplitude of the driving voltage. Namely, the amplitude of the generated force does not respond linearly to the amplitude of the driving voltage for feed-forward control of the pendulum actuator. In fact, the force amplitude remains almost unaffected by the driving voltage amplitude. Figure 7 shows three plots of generated force amplitude for three strongly differing amplitudes of the driving voltage.

This means that the pendulum actuator is capable of producing a high force amplitude even with a low driving signal. In fact, we can consider the device to be working in a sort of resonant condition where, given a small input signal, the output force maintains high amplitude values. Of course, this happens in the case for which any external disturbance on the central frame can be neglected, that means $F_e = 0$. Consequently, we can drive the pendulum with a small signal compared to the bound of saturation, and use the remainder of the driving voltage range to counteract the presence of external forces.

4. **Nonlinear kinematics and distortion of the generated forces**

The generated forces are not purely sinusoidal, because the device is not linear. The plots shown in Figure 8 show force signals in the time domain measured for different driving frequencies.

The force signals shown in Figure 8 are extracted from three different zones of the frequency range of
The zero-crossing condition of the force corresponds with the passing of the backing plates through the singularity $C_{11} = 0$. At this point, it is physically impossible to influence or to drive the force in any way. Moreover, it is also evident that the harmonic content, and consequently the degree of distortion, changes over the range of operating frequencies. To obtain a clearer understanding of the evolution of harmonic force signal content with driving frequency, the spectrum of the generated force signal was recorded for different driving signal frequencies of the pendulum actuator over the drivable range (see Figure 9).

One can see that the second, third, fifth and seventh harmonics are dominant in the resulting generated force signal. But it is possible to prove that the even harmonics are not due to the pendulating motion but rather to imperfections in the construction of the device. The main feature of an ideal pendulum actuator is that the initial angle $\alpha_0$ coincides with the unique equilibrium point at zero. In a real device implementation, however, this condition is not perfectly given due to manufacturing tolerances. Moreover, gravity force can have an influence on the position of the equilibrium point. As a consequence, the pendulating motion described by the model is superimposed by a direct transmission from the pendulum force $F$ to the generated force $F_2$. The latter effect causes the electrical frequency to be transmitted into the spectrum of the generated forces with its harmonics (these are even harmonics of the mechanical fundamental frequency). A technological improvement of the construction would lead to a reduction in the generation of even force harmonics.

The behavior of the various harmonics differs over the working frequency range. The fundamental harmonic increases continuously in amplitude, while the third and fifth harmonics each exhibit a minimum in this range. Consequently, the degree of distortion differs with the driving signal frequency. Some frequencies enhance the distortion more than others.

In this work we are interested in analyzing possible ways to reduce the distortion of the generated harmonic signal. Since the nonlinear motion of the system stems from the nonlinearities of its kinematics, it can be shown that a sinusoidal progression of $\alpha(t)$ leads to a distorted, non-sinusoidal force signal $F_2(t)$ with a strong third harmonic. So, the problem is to find the trajectory of $\alpha(t)$ which will lead to the desired reference force trajectory $F_2^*(t)$. A pure sinusoidal reference force signal at frequency $f_0$

$$F_2^*(t) = \hat{F}_2 \sin(2\pi f_0 t) \tag{11}$$
demands motion according to

\[ \alpha^*(t) = \arcsin \left( \frac{1}{m_2 l (2\pi f_0)^2} F_2^*(t) \right). \]  

(12)

In this way, it is possible to transform a desired force reference signal \( F_2^*(t) \) into a reference for the state variable \( \alpha^*(t) \). Consequently, it becomes easy to develop control algorithms based on reference trajectory tracking of the state variables.

The spectrum of the state trajectory in equation (12) can be demonstrated to be:

\[ \alpha^*(f) = \sum_{n=1}^{\infty} \frac{j B(n)}{2} [\delta(f + nf_0) - \delta(f - nf_0)] \]  

(13)

where

\[ B(n) = \sum_{j=0}^{\infty} \frac{[(2j + n - 1)!]^2}{4^{2j+n-1} [(j + \frac{n-1}{2})!]^2} (-1)^{\frac{j+n}{2}}, \]

\[ A = -\frac{\hat{F}_2}{m_2 l (2\pi f_0)^2}. \]  

(14)

The expression for \( \alpha^*(f) \) contains only odd harmonics, whose amplitudes are given by the coefficients \( B(n) \), where \( n \) is the order of the harmonic. As can be seen in the expression for \( A \), the amplitude of the harmonic components in \( \alpha \) is dependent upon the amplitude and frequency of the desired force \( F_2^* \).

Defining \( s \) as the distance between the backing plates, we obtain

\[ s = 2l \cos(\alpha). \]  

(15)

The spectrum of this signal can then be approximated by

\[ s'(f) \approx 2l[D\delta(0) + IE[\delta(f - 2\pi 2f_0 t) + \delta(f + 2\pi 2f_0 t)]] + i \sum_{i=4}^{\infty} F(i)[\delta(f - 2\pi if_0 t) + \delta(f + 2\pi if_0 t)] \]  

(16)

where

\[ D = 1 - \frac{1}{2} \sum_{n=1}^{\infty} \frac{B^2(n)}{2}, \]

\[ E = -\frac{1}{2} \left( \frac{1}{2} B^2(1) + \sum_{n=1}^{\infty} B(n)B(n + 2) \right) \]

\[ F(i) = \frac{B^2(i)}{2} + \sum_{n=1}^{\infty} B(n)B(n + i) + \sum_{n=1}^{\infty} B(n)B(i - n). \]  

(17)
Of significance is the fact that the distance $s$ has the same frequency content as the voltage $V_A$. The fundamental frequency of $s$ is referred to as the electrical frequency, while the fundamental frequency of $\alpha$ is the same as that of $F_2$ and is called the mechanical frequency. In this force generator, the electrical frequency is exactly twice the mechanical frequency. Equation (16) is necessary for determining the operating frequency range of the driving voltage for the pendulum actuator so that a voltage amplifier with a suitable frequency response can be selected.

Moreover, equation (16) shows which harmonics of the fundamental driving voltage signal are required in order to reduce the distortion, and it could be used in a feedback control loop in the case that a sensor is available for the variable $s$.

Approximating equation of motion (1) for the horizontal motion, it is reasonable to assume that the

![Graphs showing driving voltage, resulting generated force, and its spectrum.](image)

**Figure 10.** Sinusoidal driving voltage experiment: driving voltage, resulting generated force and its spectrum.
harmonic content of $s$ is iso-frequential with respect to the harmonic content of the driving voltage. Based on this consideration, we inject a certain number of harmonics of the fundamental electrical frequency into the driving voltage, as specified by equation (15), optimizing their phases and amplitudes in order to affect a reduction in the distortion of the generated force. For reference, the driving signal, the generated force and its spectrum are shown in Figure 10 for an experimental case involving a sinusoidal driving voltage:

Then, the amplitudes and phases of the second, third and fourth harmonics of the driving signal are tuned manually with the goal of minimizing the harmonic content in the resulting generated force signal. Figure 11 shows the result achieved in the experiment.

**Figure 11.** Experiment involving manual tuning of driving voltage harmonics: driving voltage, resulting generated force and its spectrum.
We notice that the distortion decreases at lower frequencies while increasing at higher ones. This is a typical behavior of systems where the attenuation of distortion is wanted. Moreover, the driving signal becomes more nervous. Continuing to inject higher frequencies in the driving signal would result in an attenuation at higher frequencies in the generated mechanical force.

In the shown experiment, we chose a driving voltage which is much lower than the maximum allowed. This was done in order to avoid exciting strong harmonic distortion due to hysteresis, a typical phenomenon of active materials such as piezoelectric ceramics. Piezoelectric materials behave increasingly linearly for decreasing amplitude of the driving voltage.

5. Limits in force generation

Not all the forces generated within the frequency range of operation can be affected by distortion reduction in the same way, due to limitations of the active material. In fact, if we consider the so-called blocking force $F_b$ of the piezoelectric material, it is possible to demonstrate that most of the pendulating motion works in a region where the force $F$ acting on the active element is greater than its blocking force. In this region the authority of the active element is low relative to the overall motion of the device.

Figure 12 shows a typical plot of generated force, in which the band of limited authority is highlighted in red.

![Figure 12](image12.png)

**Figure 12.** Limited driving authority; red regions are beyond the limits of authority of the active element.

![Figure 13](image13.png)

**Figure 13.** Limit force over the frequency and critical frequency.
This band of limited authority shown in Figure 12 increases in size with increasing frequency of pendulation. In order to maximize authority on the device dynamics, the generated vertical force should be smaller than a limiting force for each frequency. Figure 13 gives a comparison of the generated force obtained at full driving voltage with the limit force.

Within this context, we can define a critical frequency as the frequency where the limiting force and generated force at maximum driving voltage cross. For all the frequencies below the critical frequency we have strong authority on the dynamics, and consequently on the generated force $F_2$.

Inverting the model equations and considering the saturation limits, the device is capable of generating a perfectly sinusoidal force signal with amplitude between zero and the bounding values defined by the curves shown in Figure 14 (May et al., 2009).

As shown, sinusoidal force generation is possible, but only with smaller amplitude in comparison with the feedforward driving case, and within smaller frequency ranges. This is due to the saturation limits of the active material.

As mentioned, the preload force $F_0$ (i.e. the static component of the force $F$) shown in Figure 14 can be considered as a further tuning parameter of the device. It is possible to modify it by applying a preload on the spring $c_x$ (by acting on a screw positioned on the backing plates) or by applying a voltage offset to the active material. The preload force influences the location of the bounding curve with respect to the operating frequency and therefore expands the versatility of the considered device.

6. Summary and outlook

This paper has shown the capabilities and the potenti- alities of the pendulum actuator for harmonic force generation, particularly useful for vibration control purposes. The theoretical analysis of this device, based on its mathematical description, has then been confirmed by experimental results. In particular, the characterization of the distortion plays a fundamental role in the understanding and usage of this device, together with the limits of force generation, which can describe its working characteristics. These represent a solid basis for further development. Control algorithms need to be designed and implemented in order to increase the performance and versatility while broadening the scenario of applications. In order to support this part of development, sensory systems have to be available. In further works novel techniques like piezoelectric self-sensing will be investigated and experienced, because they allow the measurement of the state of the active material while driving it, and additional sensors to be mounted within the device architecture are not required.

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Erratum

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Generating periodic forces with the pendulum actuator.

Emanuele Grasso et al.


Experiments on chaotic vibrations of a post-buckled cantilevered beam connected by a string to an axial spring.

Shinichi Maruyama et al.

The two papers above should have been published in volume 18, issue 4, as part of the special issue on experiments in dynamics and control. In error, they were published in volume 18, issue 1 and volume 18, issue 2.

The publisher apologises for this error and the inconvenience caused to the authors, editors, and readers.